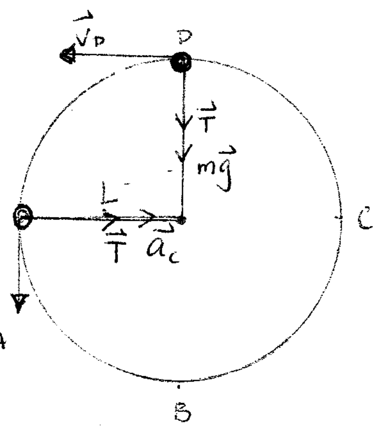
Physics 4A Exam II
Spring' 09

Name: Solutions
Lab Section: **M T**

1. (25 points) A pendulum consists of a string of length L and a bob of mass m. The string is brought to a horizontal position and the bob is given the minimum initial speed enabling the pendulum to make a full turn in the vertical plane. **(a) What is the minimum kinetic energy K of the bob? (b) What is the tension in the string when it is at the initial position?**



$W_{nc} = 0$.

energy of the system is conserved!

at point D, maximum gravitational potential energy, minimum kinetic energy.

a). $T_D + mg = m\frac{v_D^2}{L}$, with $T_D \to 0$

minimum K. when $T_D = 0$

$v_D = \sqrt{gL}$ (5)

$K_{min} = \frac{1}{2}mv_D^2 = \frac{1}{2}mgL$ (5)

b). at point A,

$T_A = ma_c = m\frac{v_A^2}{L}$

and $E_A = E_D$

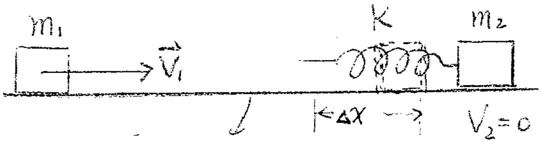
$\frac{1}{2}mv_A^2 + mgL = \frac{1}{2}mv_D^2 + mg \cdot 2L$ (10)

$\frac{1}{2}mv_A^2 = \frac{1}{2}mgL + mgL = \frac{3}{2}mgL$

$V_A^2 = 3gL$

$\therefore T = m\cdot\frac{3gL}{L} = 3mg$ (5)

2. (25 points) A mass of m_1 with initial velocity V_1 approaches another mass of m_2 initially at rest to make a head-on collision. Mass m_2 is attached by a spring of stiffness k that is initially relaxed. During their collision, **find the maximum compression of the spring** (assuming there are no energy losses because of the spring).



frictionless

during collision, momentum of the system is conserved. $P_i = P_f$ in horizontal

$$m_1 V_1 + m_2 \cdot 0 = (m_1 + m_2) V_c \qquad (10)$$

$$V_c = \frac{m_1}{m_1 + m_2} V_1$$

No energy losses, before & after collision mechanical energy of the system is also conserved.

$$E_i = E_f \quad \text{or} \quad \Delta E = 0$$

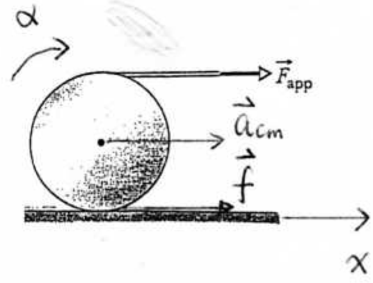
$$\frac{1}{2} m_1 V_1^2 + 0 = \frac{1}{2}(m_1 + m_2) V_c^2 + \frac{1}{2} K \Delta X^2 \qquad (10)$$

$$m_1 V_1^2 - (m_1 + m_2) \cdot \left(\frac{m_1 V_1}{m_1 + m_2}\right)^2 = K \Delta X^2$$

$$K \Delta X^2 = \frac{m_1 m_2 V_1^2}{m_1 + m_2}$$

max compression : $\Delta X = V_1 \sqrt{\dfrac{m_1 m_2}{K(m_1 + m_2)}}$ (5)

3. (25 points) A large constant horizontal force $\vec{F}_{app}$ is applied to a uniform solid cylinder of mass M and radius R by a string wrapped around the cylinder. The cylinder is rolling without slipping on the horizontal surface. **Find (a) the acceleration of the center of mass of the cylinder, and (b) the magnitude and direction of the friction force on the cylinder.**
($I = \frac{1}{2}MR^2$)



cylinder rolling clockwise with α.
the center of mass moving forward.
assume $\vec{f}$ to prevent from slipping

Newton's second Law of the center of mass:

$$F_{app} + f = Ma_{cm} \quad \text{—①} \qquad (5)$$

Rotation about an axis through the center of mass.

$$F_{app} \cdot R - f \cdot R = I\alpha = \frac{1}{2}MR^2\alpha \quad \text{—②} \qquad (10)$$

Rolling without slipping condition:

$$a_{cm} = R\alpha \quad \text{—③} \qquad (5)$$

from ②③ $\quad F_{app} - f = \frac{1}{2}MR \cdot \frac{a_{cm}}{R} = \frac{1}{2}Ma_{cm} \quad \text{—④}$

①+④ $\quad 2F_{app} = Ma_{cm} + \frac{1}{2}Ma_{cm} = \frac{3}{2}a_{cm} M$

$$\therefore a_{cm} = \frac{4F_{app}}{3M} \rightarrow$$

$$f = Ma_{cm} - F_{app} = M \cdot \frac{4F_{app}}{3M} - F_{app} = \frac{1}{3}F_{app} \rightarrow \quad (5)$$

direction of $\vec{f}$ along x

4. (25 points) A man of mass m clings to a rope ladder suspended below a large hot air balloon of total mass M (including the basket and the ladder). The balloon and the man are initially stationary relative to the ground. If the man begins to climb up the ladder at a given constant speed v relative to the ladder, **(a) in what direction and at what speed relative to the ground will the balloon move?** Any gravitational effects are not relevant in this problem. **(b) If the man then stops climbing, what is the speed of the balloon?**

a). $(m + M)$ system.

$$\cancel{\vec{F}_{\substack{net\\ext}}}^{\,0} = (m+M)\,\vec{a}_{cm} \qquad 5$$

$$\vec{a}_{cm} = 0$$

$$\vec{a}_{cm} = \frac{d\vec{V}_{cm}}{dt}$$

$$\vec{V}_{cm} = \text{constant} = 0$$

(initially stationary)

The center of mass doesn't move.

$$V_{cm} = \frac{mV_{mg} + MV_{bg}}{m+M} = 0$$

equation: $\therefore\ mV_{mg} + MV_{bg} = 0$ 10

relative velocity: $V_{mg} = V_{mb} + V_{bg} = V + V_{bg}$

$$m(V + V_{bg}) + MV_{bg} = 0$$

$$\downarrow\ V_{bg} = -\frac{mV}{m+M}\ \uparrow \qquad 5$$

the balloon moves downward direction if the man climbs up

b). when the man stops. $V=0$. then balloon stop