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Give complete solutions to the following problems. Be sure to provide all the necessary steps to support your answers.

1. Use the Gram-Schmidt process to find an orthonormal basis for  $\mathbf{W}$ .

a.  $W = \text{span}\{(1, 0, -1), (2, -2, 3)\}$

b.  $W = \text{span}\{(-1, 3, 1, 1), (6, -8, 2, -4), (6, 3, 6, -3)\}$

2. Let  $\mathbf{A} = \begin{bmatrix} 2 & 4 \\ -1 & -1 \\ 5 & 2 \end{bmatrix}$

a. find an orthogonal basis for the column space matrix  $\mathbf{A}$ , then use it to construct an orthogonal matrix  $\mathbf{Q}$ .

b. Show that the column space of  $\mathbf{A}$  equals the column space of  $\mathbf{Q}$

c. find an invertible matrix  $\mathbf{R}$  such that  $\mathbf{A}=\mathbf{QR}$ .

d. verify that  $\mathbf{A}=\mathbf{QR}$ .

3. Repeat problem 2 with  $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 1 & 0 \end{bmatrix}$