

Give complete solutions to the following problems. Be sure to provide all the necessary steps to support your answers.

1. Find the vector x determined by the given coordinate vector $[x]_B$ and the given basis B .

a. $B = \left\{ \begin{bmatrix} -3 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ -5 \end{bmatrix} \right\}, [x]_B = \begin{bmatrix} 2 \\ 5 \end{bmatrix}$

b. $\left\{ \begin{bmatrix} 5 \\ -3 \\ -5 \end{bmatrix}, \begin{bmatrix} -3 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ -5 \\ 5 \end{bmatrix} \right\}, [x]_B = \begin{bmatrix} 1 \\ 2 \\ -2 \end{bmatrix}$

2. Find the coordinate vector $[x]_B$ of x relative to the given basis $B = \{b_1, b_2\}$

$$b_1 = \begin{bmatrix} -2 \\ 3 \end{bmatrix}, \quad b_2 = \begin{bmatrix} 1 \\ 4 \end{bmatrix}, \quad x = \begin{bmatrix} 1 \\ -7 \end{bmatrix}$$

3. Find the coordinate vector $[x]_B$ of x relative to the given basis $B = \{b_1, b_2, b_3\}$

$$b_1 = \begin{bmatrix} 1 \\ -1 \\ -2 \end{bmatrix}, \quad b_2 = \begin{bmatrix} -4 \\ 5 \\ 8 \end{bmatrix}, \quad b_3 = \begin{bmatrix} 2 \\ -2 \\ 8 \end{bmatrix}, \quad x = \begin{bmatrix} -9 \\ 10 \\ -6 \end{bmatrix}$$

4. Find the change-of-coordinates matrix from B to the standard basis in \mathbb{R}^2

$$B = \left\{ \begin{bmatrix} -4 \\ 1 \end{bmatrix}, \begin{bmatrix} -3 \\ 5 \end{bmatrix} \right\}$$

5. Use an inverse matrix to find $[x]_B$ for the given x and B .

$$B = \left\{ \begin{bmatrix} 3 \\ 4 \end{bmatrix}, \begin{bmatrix} 5 \\ 7 \end{bmatrix} \right\}, \quad x = \begin{bmatrix} 5 \\ -5 \end{bmatrix}$$

6. The set $\{1-t^2, -2t-t^2, 1+t-t^2\}$ is a basis for set of polynomials P_2 . Find the coordinate vector of $p(t) = -6 - 12t + t^2$ relative to B .