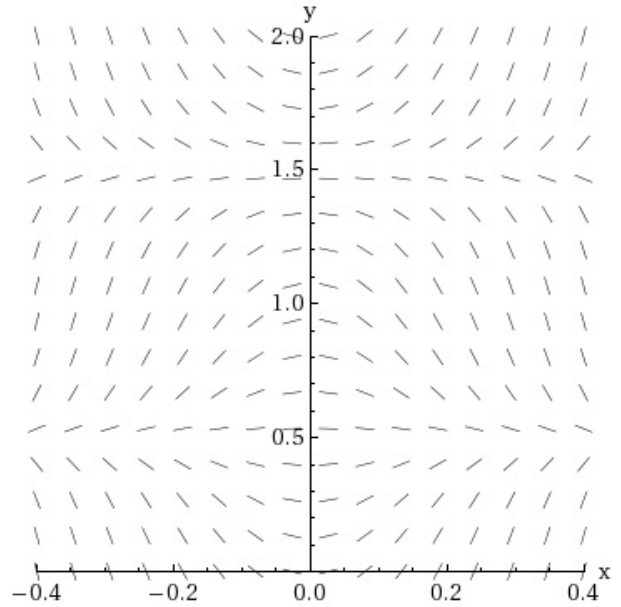


DIRECTIONS To receive full credit, you must provide complete legible solutions to the following problems in the space provided. Transfer all your answers to the space provided on the test paper.

1. A direction field for the differential equation $y' = 25x \cos(\pi y)$

a. Sketch the graphs of the solutions that satisfy the given initial conditions.

- (i) $y(0) = 0$ (ii) $y(0) = 0.5$
- (iii) $y(0) = 1$ (iv) $y(0) = 1.6$

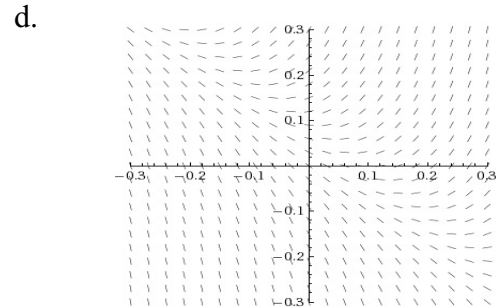
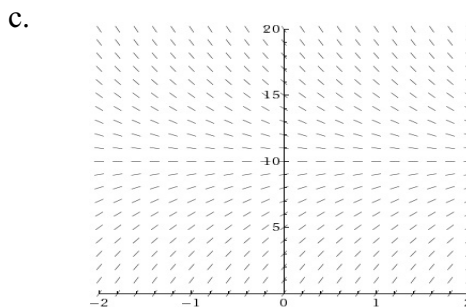
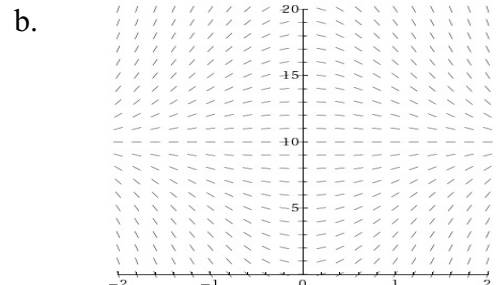
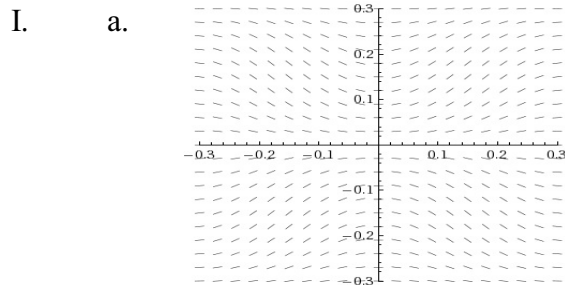


b. Find all the equilibrium solutions.

Ans _____

2. Match the differential equation with its direction field.

$y' = 10(x + y) - 1$



II. Sketch a solution curve on the correct direction field that passes through the point $(-0.1, 0.1)$

3. Use Euler's method with step size 0.5 to compute the approximate y-values y_1, y_2, y_3 and y_4 of the solution of the initial-value problem

$$y' = y - 5x, \quad y(4) = 1.$$

$$y_1 =$$

$$y_2 =$$

$$y_3 =$$

$$y_4 =$$

4. Use Euler's method with each of the following step sizes to estimate the value of $y(1)$, where y is the solution of the initial-value problem $y' = y, y(0) = 9$.

a. $h = 0.25$ and hand computation Ans _____

b. $h = 0.1$ Use calculator per class notes. Ans _____

5. The figure shows a circuit containing an electromotive force, a capacitor with a capacitance of C farads (F), and a resistor with a resistance of R ohms (Ω). The voltage drop across the capacitor is Q/C , where Q is the charge (in coulombs, C), so in this case Kirchhoff's Law gives

$$RI + \frac{Q}{C} = E(t), \text{ but } I = \frac{dQ}{dt}, \text{ so we have } R \frac{dQ}{dt} + \frac{1}{C} Q = E(t)$$

Suppose the resistance is 20 Ω , the capacitance is 0.05 F, and a battery gives a constant voltage of 80 V.

a. What is the limiting value of the charge? Ans _____

b. If the initial charge is $Q(0) = 0$ C, use Euler's method with step size 0.1 to estimate the charge, Q after half a second. Ans _____