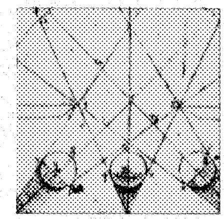
GEOMETRIC CONSTRUCTIONS

Chapter 8



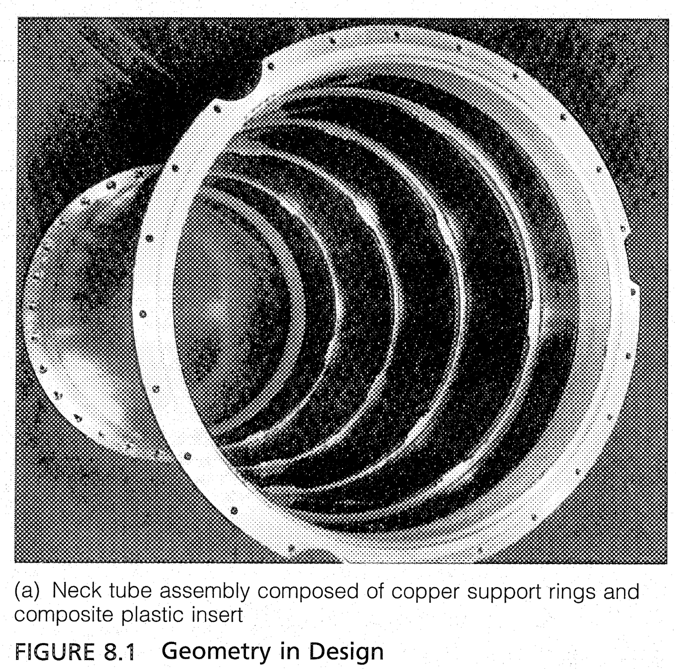
LEARNING OBJECTIVES

Upon completion of this chapter you will be able to:

1. Develop the ability to interpret graphic solutions to common geometrical problems.
2. Define and construct, via manual and CAD methods, plane geometric shapes: points, lines, curves, polygons, angles, and circles.
3. Define solid geometric shapes: polyhedra, curved surfaces, and warped surfaces.
4. Apply basic construction line drawing techniques.
5. Produce uniformly drawn and scaled examples of commonly used geometric forms and entities.
6. Employ geometric construction methods to facilitate feature locations.

8.1 INTRODUCTION

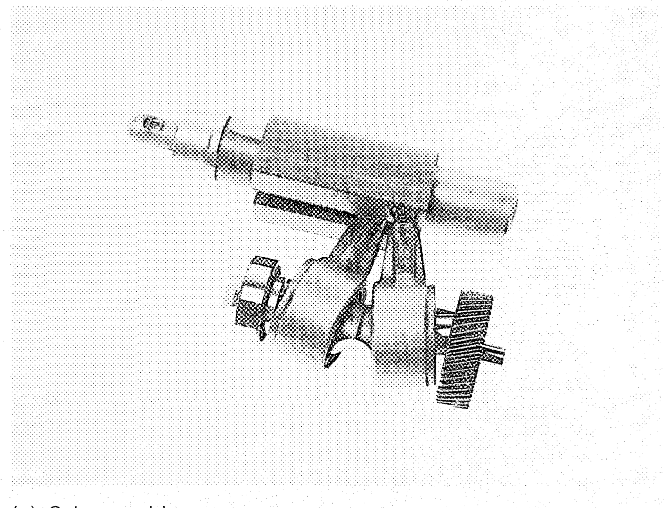
Geometric construction is a procedure for drawing figures and shapes that requires only the tools of drafting, including traditional drawing instruments and equipment and the new tools of computer hardware and software. Regardless of the tools, geometric construction requires an understanding of geometric shapes and the mechanics of their construction, as well as the ability to solve problems visually. Geometric construction emphasizes scale, uniformity of linework, and smooth joining of lines and curves when done manually. Such constructions are used extensively in industry. For example, the gravity probe assembly in Figure 8.1(a) is composed almost entirely of circular-cylindrical shapes; the stairway in Figure 8.1(b) is a cylindrical helix; and the mechanical subassembly in Figure 8.1(c) uses a variety of geometric shapes.



(a) Neck tube assembly composed of copper support rings and composite plastic insert

FIGURE 8.1 Geometry in Design

(b) Helical stairway



(c) Subassembly

FIGURE 8.1 Geometry in Design—*Continued*

8.2 GEOMETRIC FORMS

Geometric forms include a wide range of shapes and figures: **squares, triangles, arcs** and **circles, solids,** and **single-curved, double-curved,** and **warped surfaces.** The following sections provide step-by-step procedures for manually constructing common geometric forms.

8.2.1 Points and Lines

Geometric forms and shapes are points connected by lines. The **point** is the primary geometric unit in graphical construction. All projections of lines, planes, surfaces, and solids can be physically located and manipulated by identifying a series of points. These points locate ends of straight lines or are placed along a curved line to establish the line in space. Since a point exists at one position in space, it is located in space by establishing it in two or more adjacent views. (**Views** of points, lines, and other shapes are introduced in Chapter 9.)

A **line** is a series of points in space, and has magnitude (length) but not width. Although a line may be located by establishing any two points and although it may have a specified length, all lines can be extended.

Lines are used to draw edges of plane surfaces and solid shapes. A **straight line** is the shortest distance between two points. The word *line* usually refers to a straight line. A line that bends is a *curve.* When two lines are in the same plane, either they are parallel or they intersect. **Parallel lines,** symbolized by //, are the same distance apart along their entire length. Lines that intersect at an angle of 90° are **perpendicular lines,** symbolized by ⊥. Figure 8.2 shows various types of lines. Geometric constructions require you to draw arcs, circles, and other *curved lines* that use specific linework techniques.

8.2.2 Polygons

A **polygon** is a planar closed figure that has three or more straight sides. A **regular polygon** has all sides of equal length and all angles of equal size. A regular polygon can be inscribed within a circle, with corners touching the circle, or it can be circumscribed about a circle, with sides touching the circle.

A **triangle** is a three-sided polygon. The sum of its interior angles always equals 180°. In Figure 8.2, the **equilateral triangle** has equal sides and equal angles and is a regular polygon. The second type of triangle is an **isosceles triangle.** It has two equal sides and two equal angles; the unequal side is the base, and the corner opposite the unequal side is the **apex,** or **vertex.** A line drawn through the apex to the base divides an isosceles triangle into two equal triangles. **Scalene triangles** have unequal angles and unequal sides.

A **quadrilateral** is a four-sided polygon. The sum of its interior angles is 360°. Figure 8.2 shows the six types of quadrilaterals. The first four quadrilaterals have opposite sides that are equal in length and are called **parallelograms.** The first parallelogram is a **square**, because all sides and angles are equal. The second parallelogram is a **rectangle** because its opposite sides are equal and its angles are all the same. The third parallelogram is a **rhombus.** It has four equal sides and its opposite angles are equal. The fourth

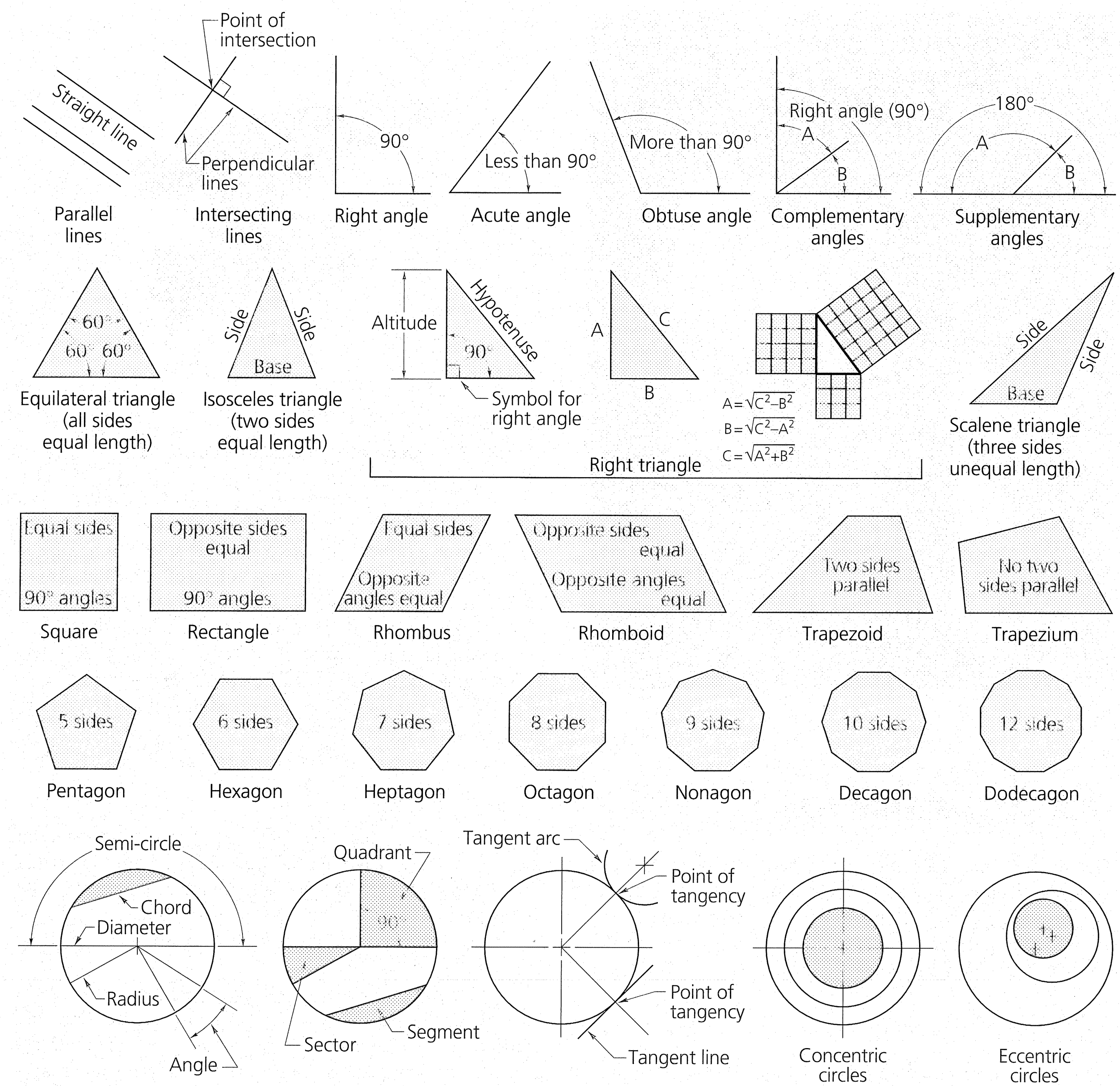


FIGURE 8.2 Geometric Shapes and Items

parallelogram, a **rhomboid**, has opposite sides parallel and opposite angles equal. A **trapezoid** has two sides parallel. When a quadrilateral has no two sides parallel it is called a **trapezium.**

Figure 8.2 also includes seven other regular polygons: **pentagon** (five sides), **hexagon** (six sides), **heptagon** (seven sides), **octagon** (eight sides), **nonagon** (nine sides), **decagon** (ten sides), and **dodecagon** (twelve sides).

8.2.3 Angles and Circles

Angles, represented by the symbol ∠, are formed by two intersecting lines (Fig. 8.3). The angle measurement of the distance between lines is typically expressed in degrees and sometimes in radians.

Various Types of Angles (Fig. 8.3)

Acute angle Less than 90°.

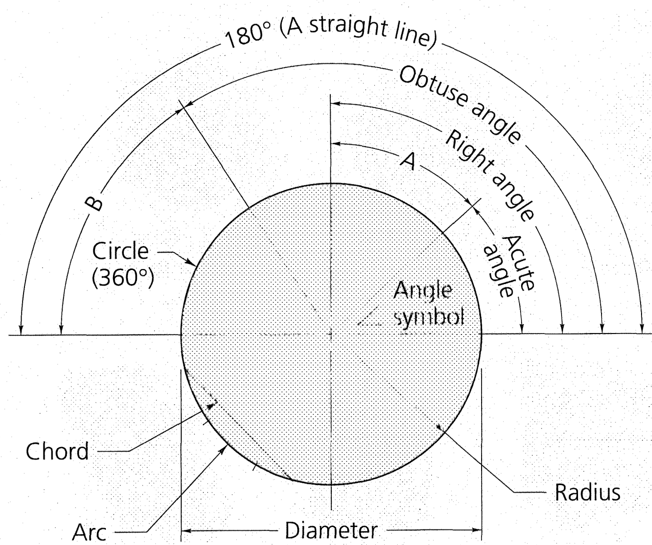


FIGURE 8.3 Angles and Circles

Right angle 90°, formed by two perpendicular lines.

Obtuse angle More than 90° but less than 180°.

180° angle A **straight line.**

Complementary angles Two angles whose sum is 90°.

Supplementary angles Two angles whose sum is 180°.

Circles represent holes and solid round shapes on drawings. A full circle is 360°.

Parts of a Circle (Fig. 8.3)

Circumference The distance around the circle.

Diameter The distance measured from edge to edge and through the center of the circle.

Radius One-half the diameter, measured from the center of the circle to the circumference.

Chord A straight line that connects two points on the circle's circumference.

Arc A continuous portion of the circumference, from one fixed point along it to another.

Concentric circles Have different radii but the same center point.

Eccentric circles Have different center points and different radii.

8.2.4 Polyhedra

Polyhedra (Fig. 8.4) are solids formed by plane surfaces. Every surface (face) of each form is a polygon. **Prisms** are polyhedra that have two parallel polygon-shaped ends and sides that are parallelograms. The **cube** is a polyhedron that has six equal sides. A **pyramid** is a polyhedron that has a polygon for a base and triangles with a common vertex for faces. A **tetrahedron** is a pyramid that has four equal sides. Figure 8.4 also illustrates a **right pyramid,** a **truncated pyramid,** and an **oblique pyramid.**

8.2.5 Curved Surfaces

Curved surfaces are divided into two categories: **single curved** (also called **ruled surfaces**) and **double curved.** Forms that are bounded by single-curved surfaces include **cones** and **cylinders.** Variations of cones include the **right cone,** the **frustum of a cone,** the **oblique cone,** and the **truncated cone.** Figure 8.4 also shows a **right cylinder** and an **oblique cylinder.** Double-curved surfaces (Fig. 8.4) are generated by moving a curved line about a straight-line axis and include a **sphere,** a **torus,** and an **ellipsoid.**

Figure 8.5(a–e) presents examples of solid shapes used in the parametric design of parts. Can you see the prism, cylinder, and sphere shapes and their intersections? The parametric solid model of the shaft in Figure 8.5(c) and its accompaning drawing in (d) provide an example of a part design created almost entirely out of cylinders (revolved solids). The parametric model in Figure 8.5(e) is an example of an unfolded flat pattern of a sheet metal part. The part's solid shapes are all prisms except for the hole (circle) being added to the model at this stage in the design.

8.3 DRAWING GEOMETRIC CONSTRUCTIONS

The following sections present step-by-step instructions for drawing geometric constructions, including parallel lines, perpendicular lines, angles, circles, polygons, tangencies, tangent arcs, curves, conics, involutes, spirals, and helices.

8.3.1 Drawing Parallel and Perpendicular Lines

Parallel and perpendicular lines are easily constructed using a straightedge and a triangle. In Figure 8.6 an adjustable triangle is used in the construction, but it could be any triangle. *Position 1* is the first line drawn. Use the following steps.

1. Move the triangle along the straightedge to *position 2* and draw a parallel line.
2. Rotate the triangle to *position 3.* Then draw a line perpendicular to the first two lines, on the same edge of the triangle that you used before it was rotated.

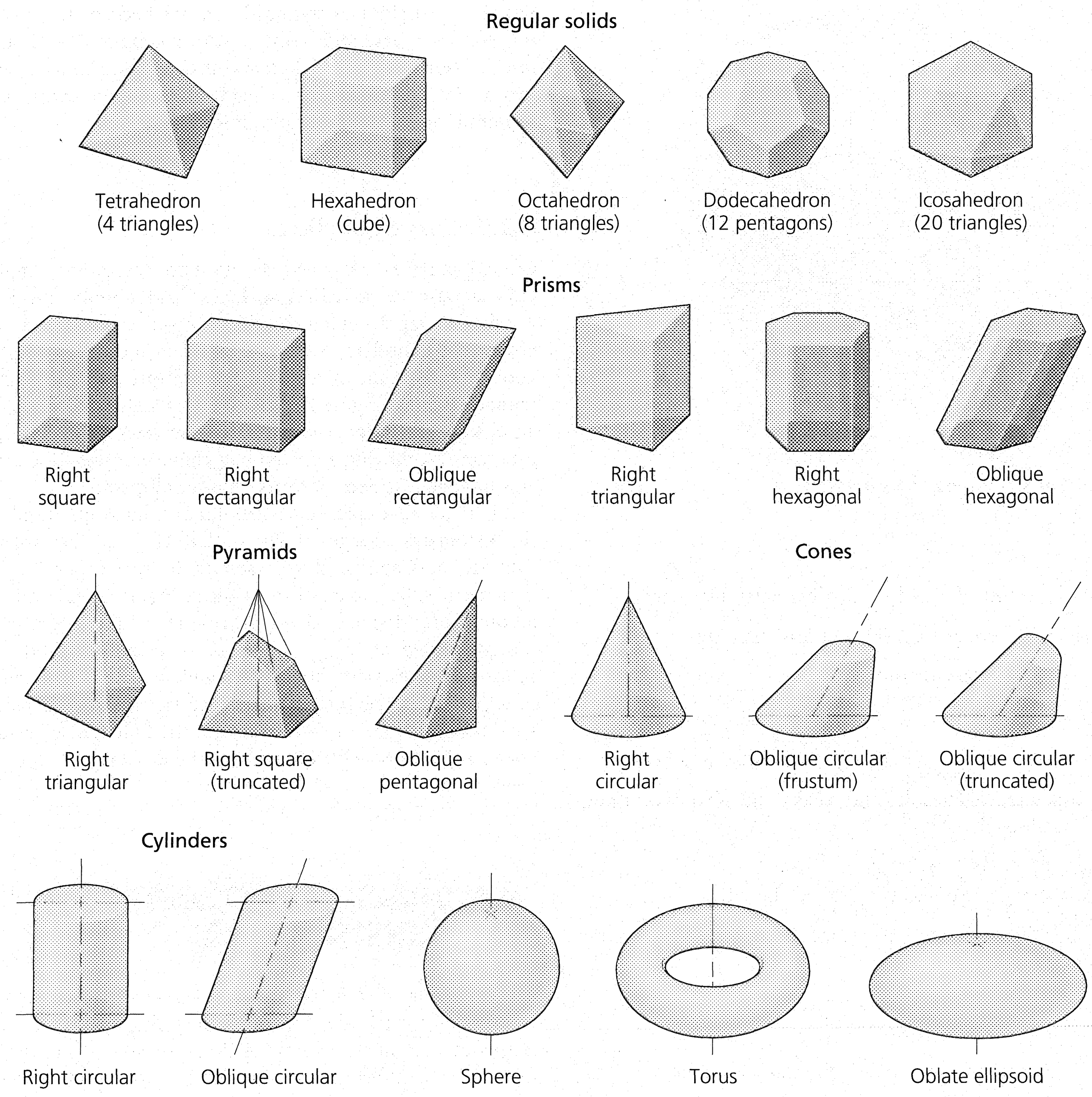
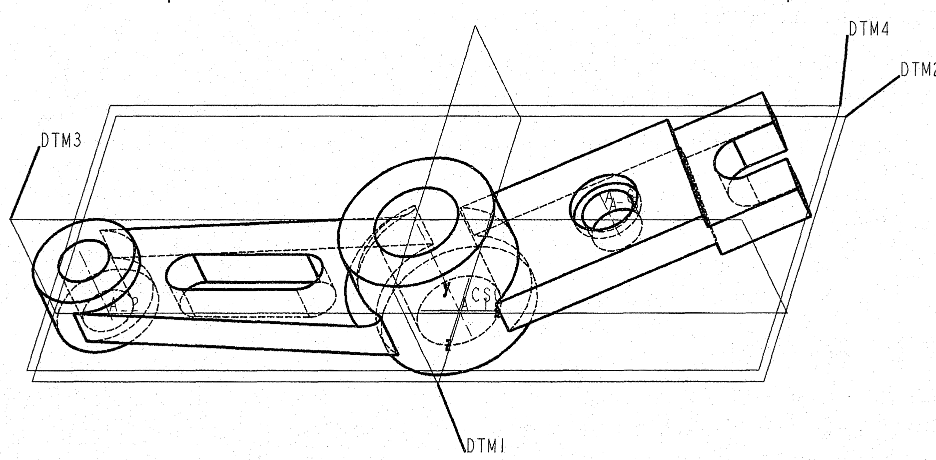
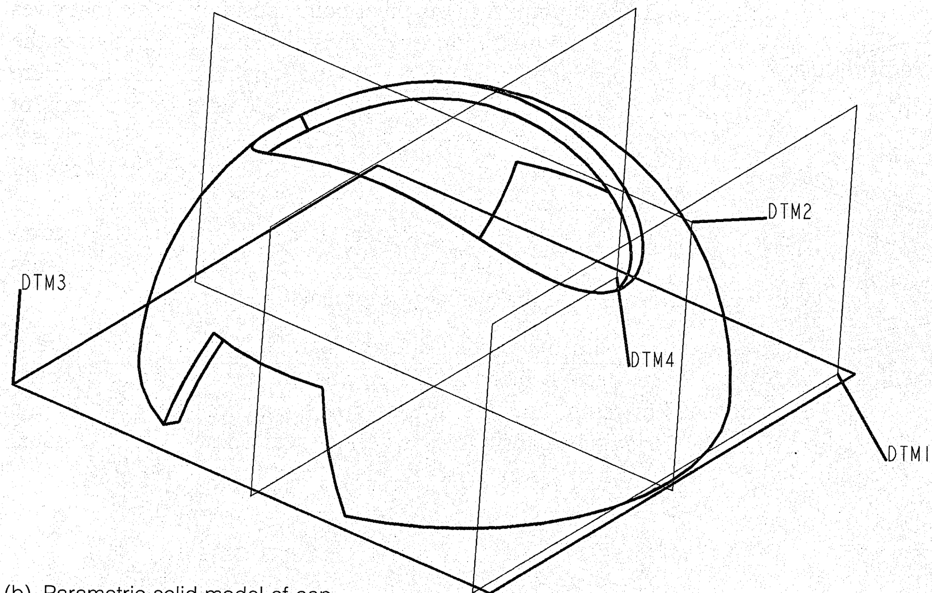


FIGURE 8.4 Solids

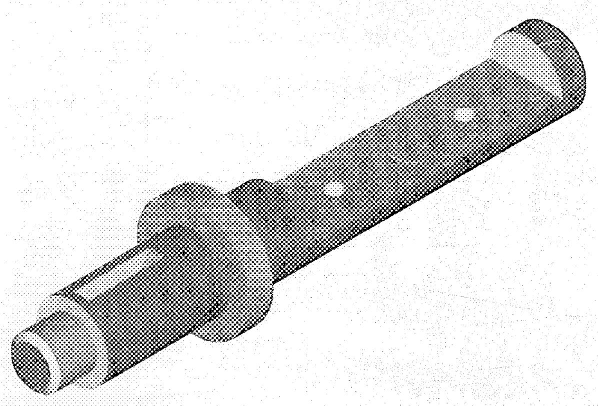


(a) Parametric solid model of arm

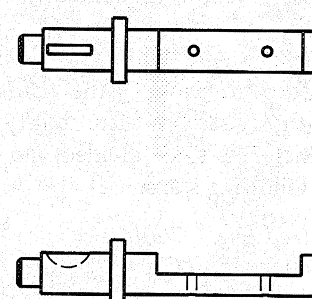
FIGURE 8.5 Solid Shapes and Modeling



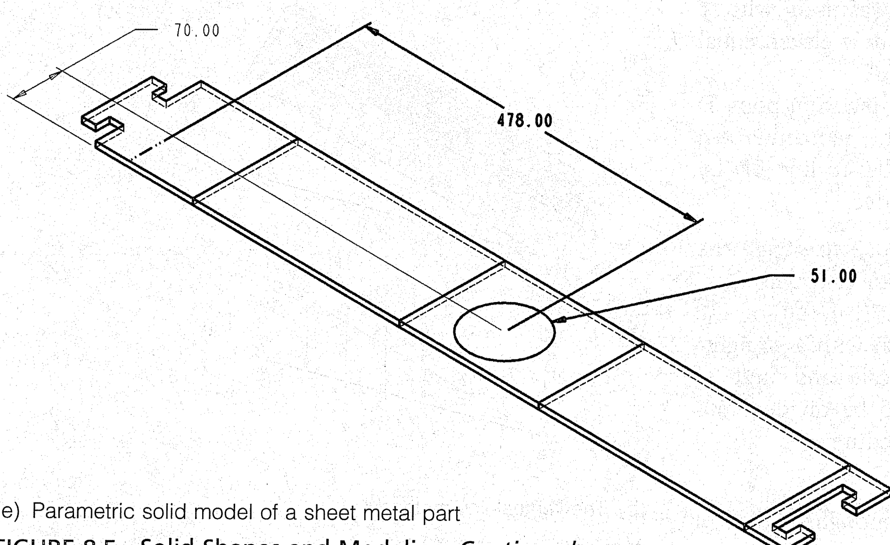
(b) Parametric solid model of cap



(c) Parametric solid model of shaft



(d) Parametric drawing of shaft



(e) Parametric solid model of a sheet metal part

FIGURE 8.5 Solid Shapes and Modeling–*Continued*

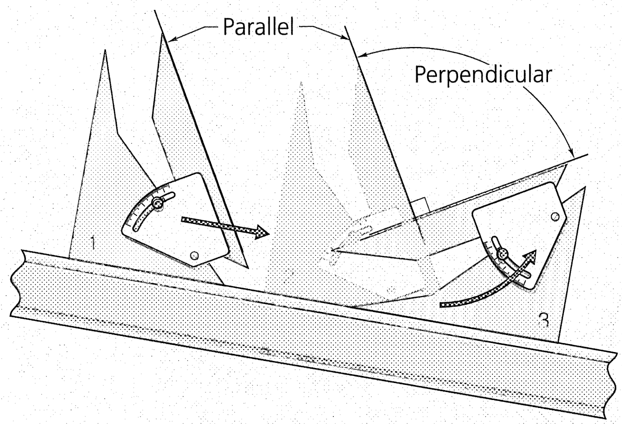


FIGURE 8.6 Drawing Parallel and Perpendicular Lines

8.3.2 Dividing a Line into Equal or Proportional Parts

One way to divide a line is to calculate its length, divide the length by the number of required parts, and then use the result to mark the divisions. However, this method accumulates error. A better way to divide a line equally or proportionally is by one of the following methods.

Figure 8.7(a) illustrates the **parallel-line method** for dividing a line equally. Any type of scale and unit of measurement will do. To simplify the construction and measuring, choose the most convenient unit type and scale. In Figure 8.7(a), line AB is to be divided into eleven equal segments. Use the following steps.

Parallel-Line Method [Fig. 8.7(a)]

1. Draw a construction line AC that starts at either end of line AB. This line is any convenient length (slightly longer works well). Angle A should not be less than 20° or more than 45° or it will be hard to project the divisions from the construction line AC to the original line AB.
2. Find a scale that will divide line AB into approximately the number of parts needed, and mark these divisions on line AC. Here, the full-size inch scale was used, with $\frac{1}{2}$ inch marking each division. There are now eleven equal divisions from A to D that lie on line AC.
3. Set the triangle to draw a construction line from point D to point B. Then through each of the remaining ten divisions draw construction lines parallel to line BD by moving the triangle along the straightedge.

It is also possible to use dividers for step 2 to divide the construction lines into the required number of equal parts.

In the **vertical-line method** [Fig. 8.7(b)], all of the projection lines are vertical and are drawn with a straightedge and any triangle. Any type of scale and unit of measurement can be used. Line AB is to be divided into seven equal parts. Use the following procedure.

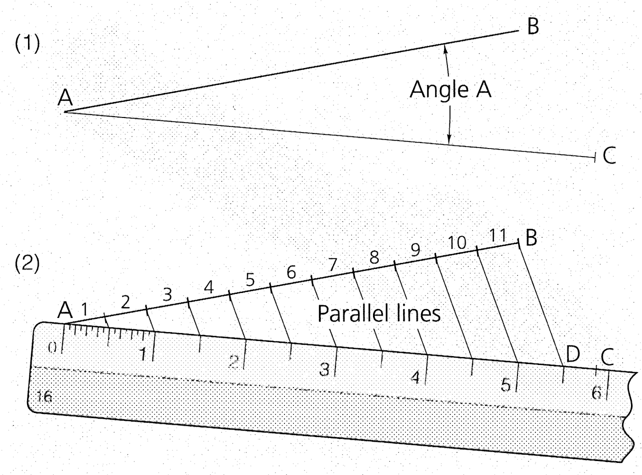
Vertical-Line Method [Fig. 8.7(b)]

1. Draw a vertical construction line BC through point B of line AB.
2. With point A as the pivot point, position a scale that gives the required number of divisions and equally divides the distance from point A to some point on line BC. Here full-scale U.S. customary units were used—a $\frac{1}{2}$ in. unit of the scale corresponds to each division—to mark points 1 through 7. It is necessary to use a scale whose overall length of seven units is longer than the line AB.
3. Using the vertical side of a triangle, draw construction lines from points 1 through 7 to line AB. This establishes seven equally spaced segments along line AB.

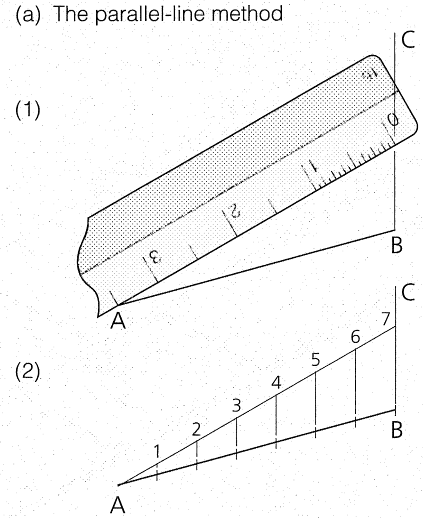
With the vertical-line and parallel-line methods, there is no need to make measurements that are less than one easily measured unit, regardless of the mathematical value of the resulting divisions. The scale serves only to measure equal units; any scale that will do so can be used.

8.3.3 Proportional Division of a Line

You may occasionally need to divide a line into parts that are in a specified proportion to each other. For instance, if a line must be divided so the first part is three times as long as the



(a) The parallel-line method



(b) The vertical-line method

FIGURE 8.7 Dividing a Line into Equal Parts

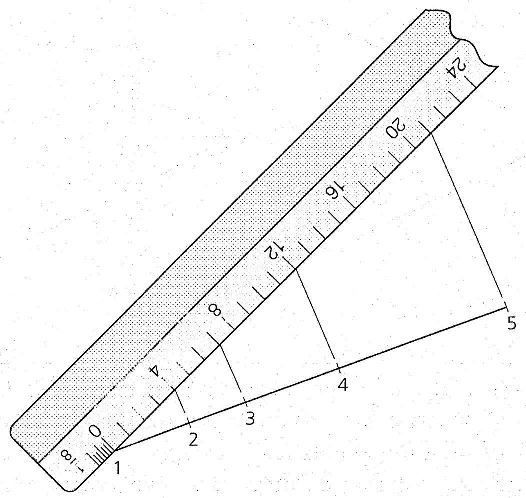


FIGURE 8.8 Proportional Division of a Line

second part, this ratio is written as 3:1 and is read as "three parts to one part" or "three to one." To divide a line proportionally, you use a method similar to that for dividing a line into equal parts.

Figure 8.8 illustrates dividing a line into four parts that have proportions of 4:3:5:9. The following steps are used.

1. Add the proportions of the parts: 4 + 3 + 5 + 9 = 21. This is the number of equal parts that are to be measured on the scale.
2. Draw a construction line at an angle to, and longer than, the given line. Set the scale to make 21 equal divisions. Make the first mark at 4 units, add 3 units, and make the second mark at 7 units; add 5 more units and make the third mark at 12 units. Add 9 units to bring the total to 21 units.
3. Project these marks to position points 2, 3, 4, and 5 on the given line using the parallel-line method. This creates line segments in proportions of 4:3:5:9.

8.3.4 Bisectors for Lines and Angles

A **perpendicular bisector** of a line *divides that line into two equal parts.* A perpendicular bisector can be constructed with only compass and straightedge (Fig. 8.9) by using the following steps.

Bisecting a Line (Fig. 8.9)

1. Set the compass at radius (R) equal to a distance greater than one-half of AB.
2. Using points A and B as centers, draw intersecting arcs to establish intersection points 1 and 2.
3. Draw construction line 1-2 by connecting the two new points. Line 1-2 intersects line AB at its midpoint and is perpendicular to it.

Figure 8.10 shows how to *divide an angle into two equal parts.* Lines AB and BC intersect and form angle ABC. Use the following steps.

Bisecting an Angle (Fig. 8.10)

1. Set the compass to any convenient radius. For small angles and short lines, extend the lines that form the angle. With point B (vertex) as the center, draw an arc (radius R) that locates points 1 and 2. The length of B1 is equal to B2.
2. Using the radius R, draw arcs from points 1 and 2. Point 3 is the intersection of these two arcs.
3. Draw line 3B. This is the bisector of the angle.

8.3.5 Locating the Center of a Known Circle

The perpendicular bisector of a chord of a circle passes through the center of the circle. If a significant portion of a circle is known, its center can be located by establishing the perpendicular bisectors of any two chords of the circle. In Figure 8.11, chords 1-2, 2-3, and 3-1 form a triangle inside the circle. Bisectors of two of these chords cross at the center of the circle (point C). The third perpendicular bisector, though not necessary, serves as a check.

8.3.6 Constructing a Circle Through Three Given Points

Using the procedure for constructing a perpendicular bisector of a line, you can construct a circle through three given

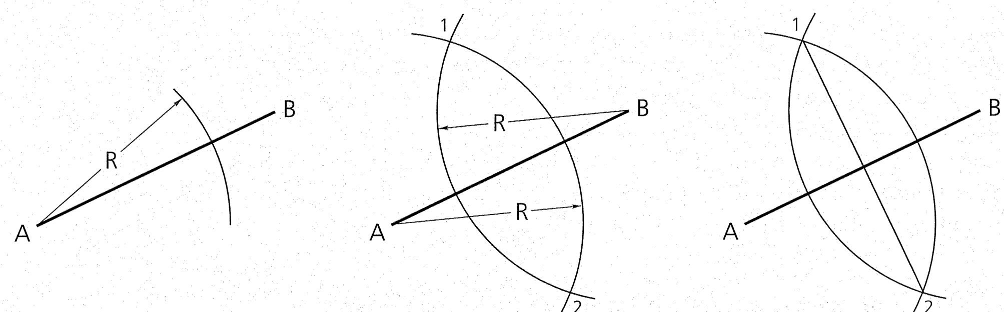


FIGURE 8.9 Bisecting a Line **1** **2** **3**

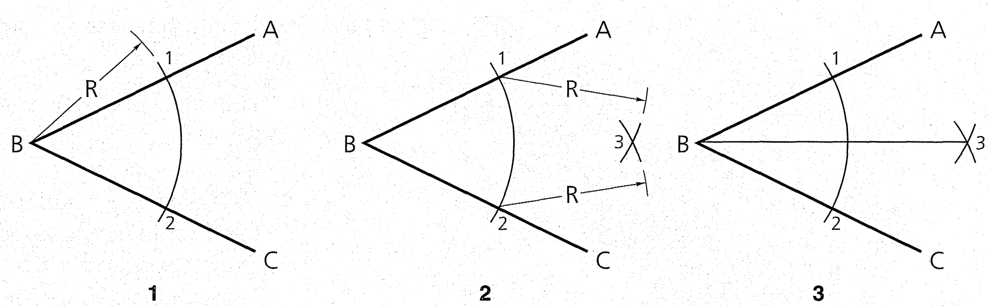


FIGURE 8.10 Bisecting an Angle

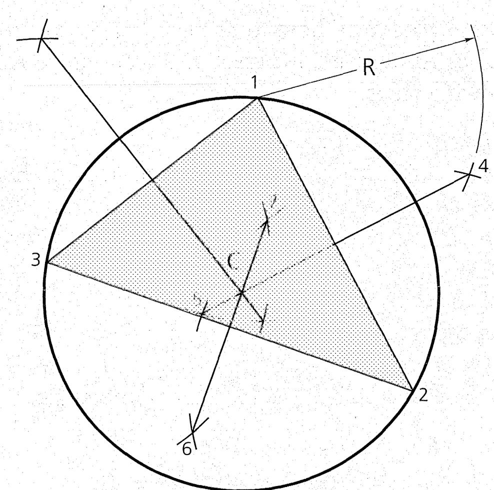


FIGURE 8.11 Finding the Center of a Circle

points in space. Figure 8.12 shows points 1, 2, and 3. The following steps were used.

1. Connect the three points with lines, and then construct perpendicular bisectors for any two chords of the circle (lines 1-2 and 1-3 here). The perpendicular bisectors intersect at the center of the required circle (C).
2. Draw the circle using as the radius the distance from C to any of the three points (C-1, C-2, or C-3).
3. Check the solution by drawing a perpendicular bisector through chord 2-3.

8.3.7 Inscribed Circle of a Triangle

An **inscribed circle of a triangle** is a circle that is tangent to (touches) each side of the triangle. Figure 8.13 illustrates the procedure for constructing the inscribed circle of a triangle. The given triangle is represented by points 1-2-3. The following steps were used.

1. Find the center of the circle by bisecting a minimum of two of the triangle's angles. The angle at point 1 is bisected by drawing arc RA to establish points 4 and 5 (RA is any convenient length).
2. From points 4 and 5, draw equal arcs (RB). Point 6 is the intersection of the two arcs.
3. Draw line 1-6 to establish the bisector of the angle, and extend this line beyond point 6.
4. Bisect the angle at point 2 by drawing arc RC to locate points 7 and 8.

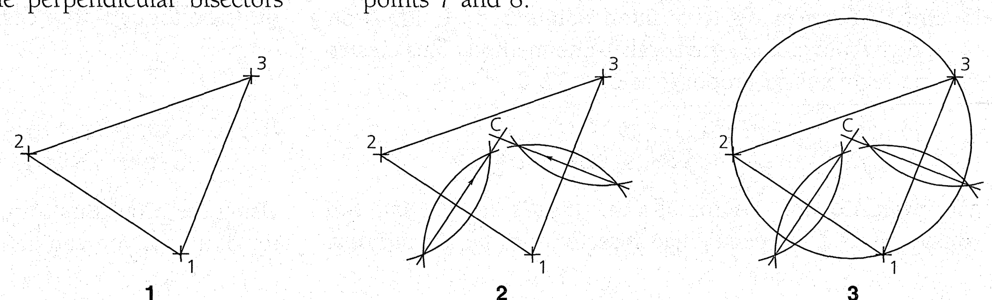


FIGURE 8.12 Drawing a Circle Through Three Given Points

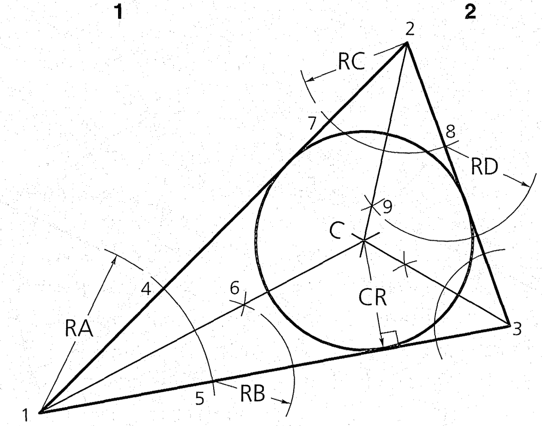


FIGURE 8.13 The Inscribed Circle of a Triangle

5. Establish point 9 by drawing equal arcs (RD) from points 7 and 8.
6. Draw a line from point 2 through point 9, and extend it to intersect the first bisector. The intersection of these two lines determines the center of the circle (C). To check the accuracy of this point, construct a third bisector that will also meet at point C.
7. Draw a line from point C perpendicular to one of the triangle's sides (side 1-3 here) to determine radius CR.
8. To complete the solution, draw the inscribed circle using C as the center and distance CR as the radius.

8.3.8 Circumscribed Circle of a Triangle

A **circumscribed circle of a triangle** touches the three vertex points of a triangle. Constructing a circumscribed circle of a triangle uses the method for constructing a circle through three given points. In Figure 8.11, the perpendicular bisectors of sides 1-2, 2-3, and 1-3 have been drawn. The intersection of the perpendicular bisectors 4-5 and 6-7 establish the center of the circle (C). A third perpendicular bisector (of line 1-3) can be drawn to check for accuracy. The radius of the circle is the distance from C to any of the three points on the triangle (C-1, C-2, or C-3).

8.3.9 Drawing a Triangle with Sides Given

Drawing a triangle, given the sides, is called **triangulation.** Use lines A, B, and C in Figure 8.14 to construct a triangle via the following steps.

1. A, B, and C are given.
2. Draw the baseline (C) and swing an arc as shown.
3. Swing arc B from the end of line C. The intersection of arcs A and B determines the vertex of the triangle.
4. Draw lines A and B.

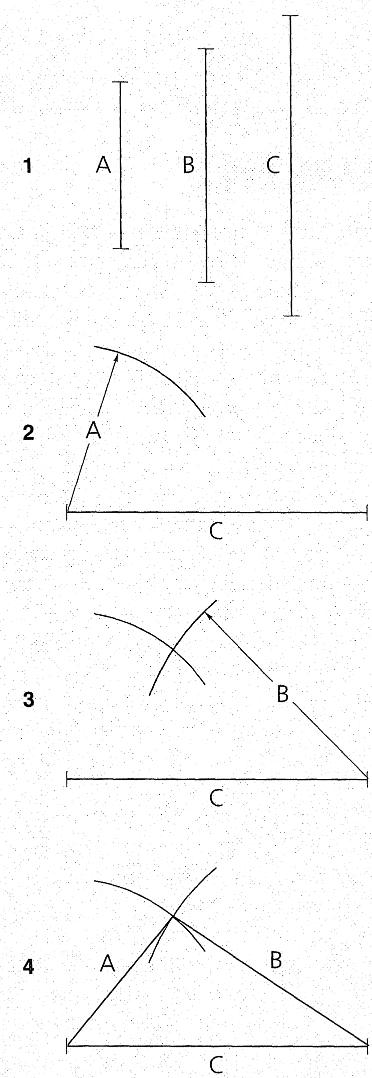


FIGURE 8.14 Drawing a Triangle with Sides Given

8.4 REGULAR POLYGON CONSTRUCTION

Polygons are closed figures having three or more sides. **Regular polygons** have equal-length sides and equal angles. All regular polygons can be *inscribed* within a circle and can also be *circumscribed* about the outside of a circle. (In this case, the circle drawn tangent to the polygon's sides is inscribed in the polygon.)

To draw a particular regular polygon, you must have at least one dimension. The two dimensions that are used for even-number-sided figures (squares, hexagons, octagons, etc.) are *across corners* and *across flats*. Across corners is the maximum measurable straight-line distance across the figure and is equal to the diameter of its circumscribing circle. Across flats is the minimum measurable straight-line distance across the figure and is equal to the diameter of its inscribed circle.

8.4.1 Constructing an Equilateral Triangle

The simplest type of regular polygon is the **equilateral triangle,** which has three equal sides and three equal angles (60°). Figure 8.15 shows the steps used to construct an equilateral triangle.

1. Given the length of one side of the triangle (line 1-2), draw the baseline.
2. From endpoints 1 and 2, draw arcs using the side length as the radius. The intersection of the arcs establishes the vertex of the triangle

Another way to construct an equilateral triangle is to lay out the baseline and then use a 60° triangle to draw each side [Fig. 8.15(3)]. The intersection of the sides establishes the vertex.

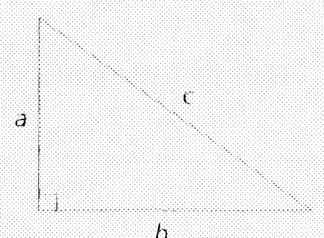
Focus On . . .

GEOMETRY

If you want to study the properties of figures and the relationships between points, lines, angles, surfaces, and solids, you study *geometry*. Geometry actually means "earth measurement." The name can be traced back to the way in which people first used those concepts. Practical geometry grew out of the needs of the Egyptians to survey their land to reestablish land boundaries after periodic flooding of the Nile. The flooding itself left a rich and sought-after soil. The workers who made these measurements became known as "rope stretchers" because they used ropes to do their measuring. The Egyptians also relied on geometry to help build their temples and pyramids.

Around 600 B.C., the Greeks returned from their travels through Egypt and brought with them their first knowledge of geometry. Thales, the most famous of those returning from Egypt, was the first to show the truth of a geometric relationship by showing that it followed in a logical and orderly fashion from a set of universally accepted statements (axioms or postulates). You may remember axioms and proofs from your geometry class.

Thales's student, Pythagoras, established a society in Italy that was devoted to the study of geometry and arithmetic. His most famous work was the theorem that bears his name. His influence was felt for centuries. Every student of geometry and trigonometry knows the Pythagorean theorem, which relates the lengths of the three sides of a right triangle ($a^2 + b^2 = c^2$). The side opposite the right angle is the longest side, or the hypotenuse (c). The other two sides, a and b, are the sides opposite the other two angles in the triangle.

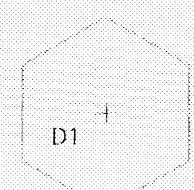


The Pythagorean theorem. In a right triangle with sides *a* and *b* and hypotenuse *c*, $a^2 + b^2 = c^2$.

While the early Greeks and others were able to make great contributions, it was not until 1796 that a great advancement was made in geometry. A 19-year-old German, Carl Friedrich Gauss, proved it was possible to construct a regular 17-sided polygon using a compass and a rule. The Greeks had only been able to construct regular polygons of 3, 4, 5, 6, 8, 10, and 15 sides. The 17-sided polygon was a major breakthrough. In 1799, Gauss was awarded a Ph.D. for developing the first proof of the fundamental theorem of algebra.

It may be difficult to appreciate how each contribution to geometry made engineering graphics possible. Even the most sophisticated CAD system makes use of fundamental geometric principles that were developed long ago. Operators of modern CAD systems can use the **POLYGON** command to create regular polygons of any number of sides (17 sides or 1000 sides, for example).

What started out as a way to measure the earth evolved into a discipline that is the key to solving most engineering problems. All the geometric constructions used today in drafting were developed by individuals building on previous developments of others. No doubt, humankind will continue building on the past for the future.



Command: POLYGON
Number of sides: 8
Edge of/<Center of Polygon>: D1
Inscribed in circle/Circumscribed about circle: C
Radius of circle: .75

The POLYGON command using an AutoCAD system.

8.4.2 Constructing a Square

A **square** has four equal sides and four equal angles. Figure 8.16 shows a square inscribed within a circle; the circle's diameter equals the distance across the inscribed square's corners. Figure 8.16 also shows a circle inscribed within a square; the circle's diameter equals the side of the square. Three methods can be used to construct a square.

In Figure 8.16(1), the base is drawn using the side length. Then an arc R is drawn using point 1 as the center and line 1-2 as the length. The intersection of the arc with a vertical line extended from point 1 establishes the height of the square. The square is then completed.

In Figure 8.16(2), the baseline is drawn and a 45° triangle is used to draw lines diagonally through points 1 and 2 to establish points 3 and 4. Points 3 and 4 are at the intersection of the diagonals and lines drawn vertically through points 1 and 2.

Figure 8.16(3) uses a circle template or a compass to draw inscribed and circumscribed circles to construct a

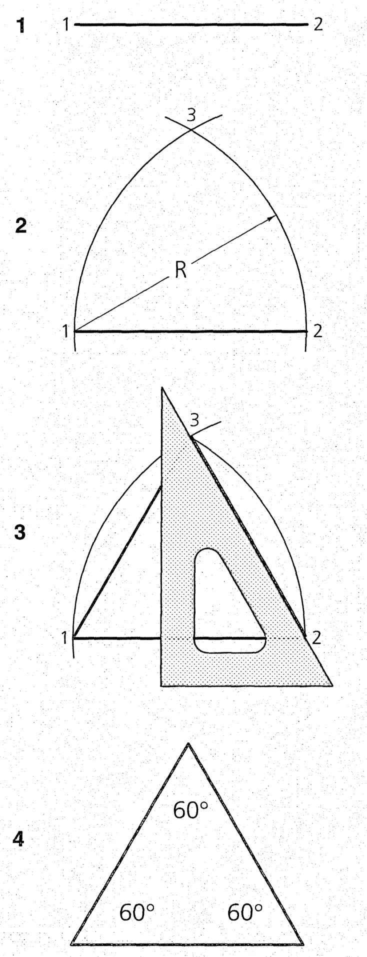


FIGURE 8.15 **Drawing an Equilateral Triangle**

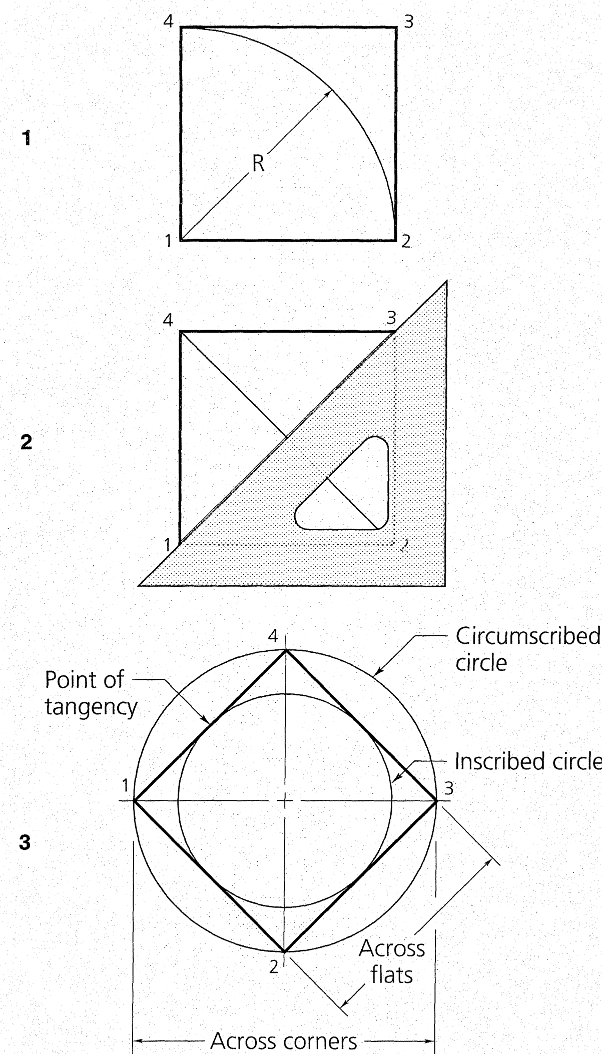


FIGURE 8.16 **Drawing a Square**

square. The point of tangency is where the square's sides touch the circumference.

8.4.3 Constructing a Pentagon

A **regular pentagon** has five equal sides and five equal angles. Figure 8.17 illustrates how to draw a pentagon (the diameter of the circumscribing circle is given).

1. Draw the centerlines of the figure, and then draw the circle. The center of the circle is point 0.
2. Find point 2 by bisecting line 0-1. Radius R (2-3) is used to establish point 4 on the vertical centerline. The distance from point 3 to point 4 (radius R2) is then used to locate point 5 on the circumference of the circle
3. Draw side 3-5. Use radius R2 from point 5 to establish point 6. Then use R2 to establish the remaining sides of the pentagon.

8.4.4 Constructing a Hexagon

A **regular hexagon** has six equal sides and six equal angles. Figure 8.18 shows how to construct a hexagon. In this figure, the distance across the flats was known. The distance across the flats is equal to the diameter of the inscribing circle. The following steps were used.

1. Locate the center of the hexagon.
2. Draw a circle equal to the distance across the flats, then, with a 30° angle, construct tangents to the circle.

If you know the distance across the corners, then draw the circumscribed circle first and mark off each side length along the circumference, using a distance equal to the radius of the circle (use dividers) [Fig. 8.19(a)]. Figure 8.19(b) shows an alternative method for producing a hexagon.

8.4.5 Constructing an Octagon

A **regular octagon** has eight equal sides and eight equal angles. To draw an octagon with the distance across the corners known, draw the circumscribed circle first and then mark off the side lengths around the circumference. If you

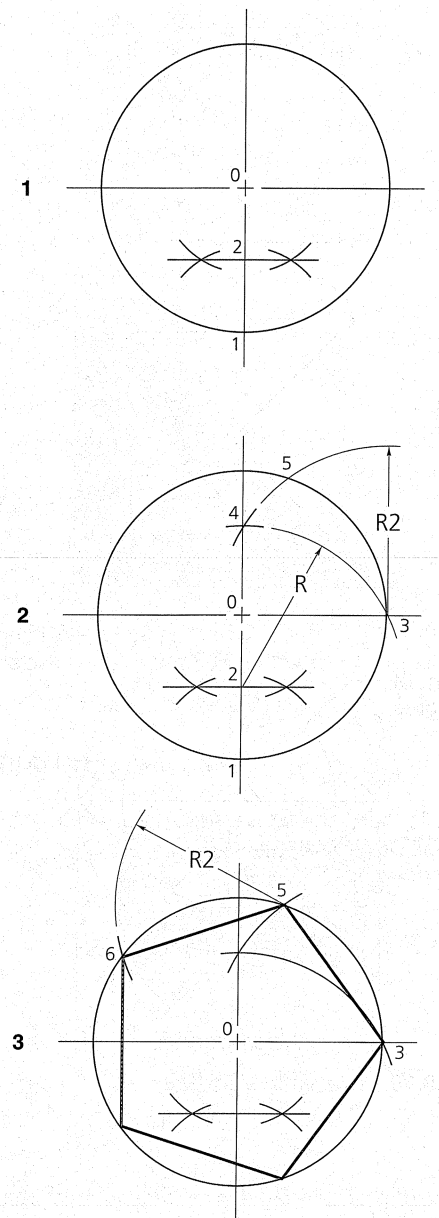


FIGURE 8.17 Drawing a Pentagon

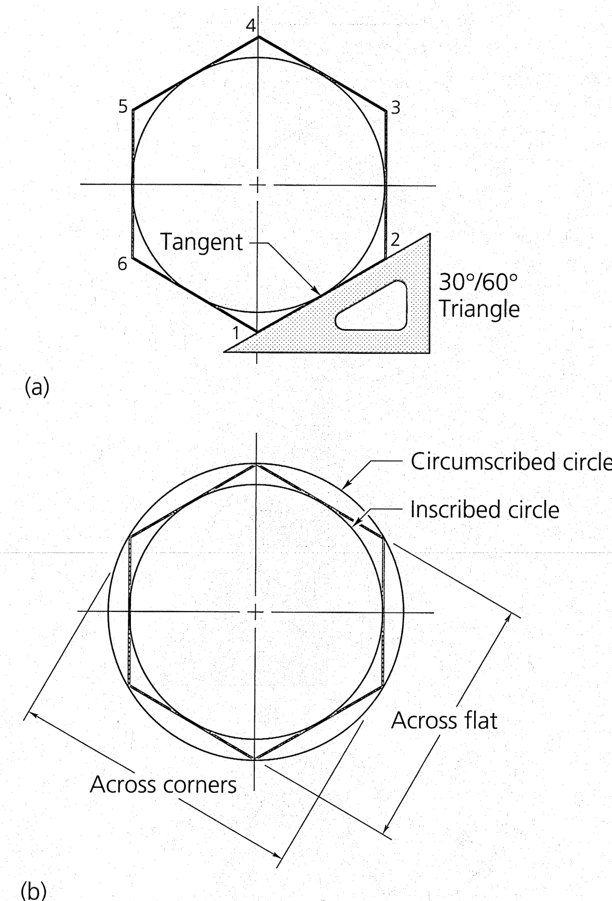


FIGURE 8.18 Drawing a Hexagon Using Inscribed and Circumscribed Circles

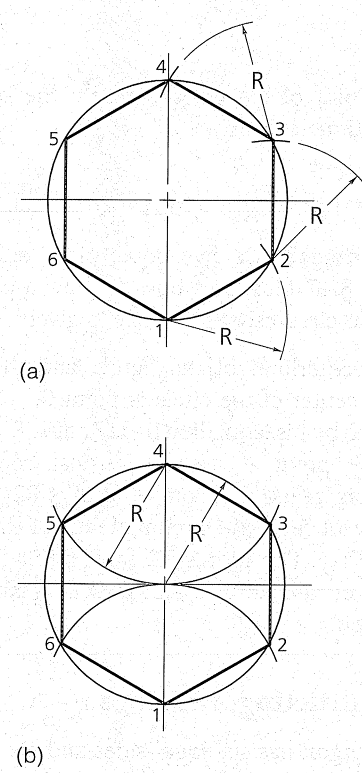


FIGURE 8.19 Drawing a Hexagon

know the distance across the flats, then use a 45° triangle to draw tangent lines to establish the eight sides (Fig. 8.20).

8.4.6 Constructing a Regular Polygon with a Specific Number of Sides

To construct a **regular polygon** with a specific number of sides, divide the given diameter of the circumscribing circle via the parallel-line method described earlier (Fig. 8.21). A polygon with seven sides is used as an example. The following steps were used.

1. Construct an equilateral triangle (0-7-8) with the diameter (0-7) as one of its sides.
2. Draw a line from the apex (point 8) through the second point on the line (point 2).

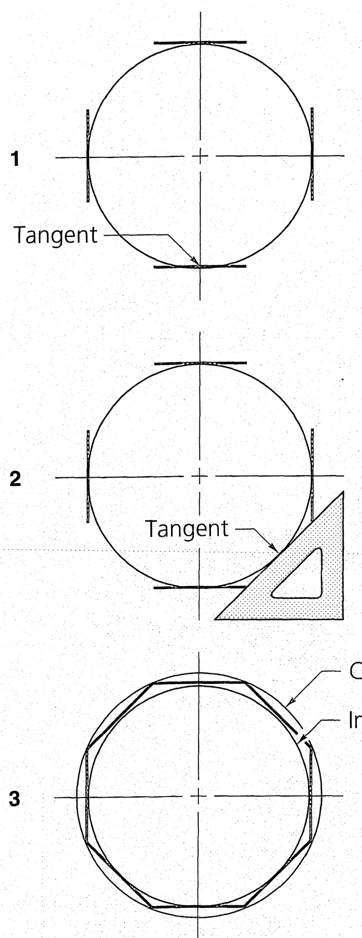


FIGURE 8.20 Drawing an Octagon

3. Extend line 8-2 until it intersects the circle at point 9. Radius 0-9 will be the size of each side of the figure.
4. Using radius 0-9, mark off the corners of the polygon, then connect the points.

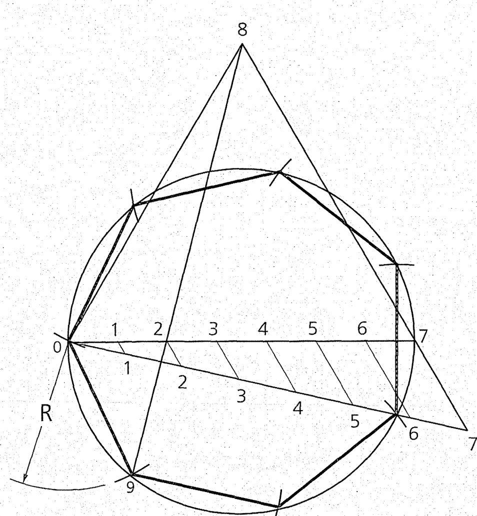


FIGURE 8.21 Drawing a Regular Polygon

8.5 TANGENCIES

An arc that touches a line at only one point is tangent to that line, and the line is tangent to the arc. Two curves can also be tangent. The line and the arc touch at only one place even if they are extended. If a line and an arc are tangent, (1) the tangent line is perpendicular to the radius of the arc at the point of tangency, and (2) the center of the arc is on a line that is perpendicular to the tangent line and extends from the point of tangency.

Figure 8.22 illustrates principle 1. To draw a line tangent to the circle at point 1, draw radius C1. Construct line AB perpendicular to the radius line C1 passing through point 1. Line AB is tangent to the circle at point 1.

Figure 8.23 illustrates principle 2. To draw a circle tangent to a given line, first project a line perpendicular to the given line AB from point T (tangent point). The center of the circle will be on this line. Locate the center point by marking off an arc from point T, using the radius of the circle. Using the same radius (line CT), draw the tangent circle.

8.5.1 Line Tangent to Two Circles

Figure 8.24 illustrates the procedure for finding the points of tangency between a line and two circles. A line can be

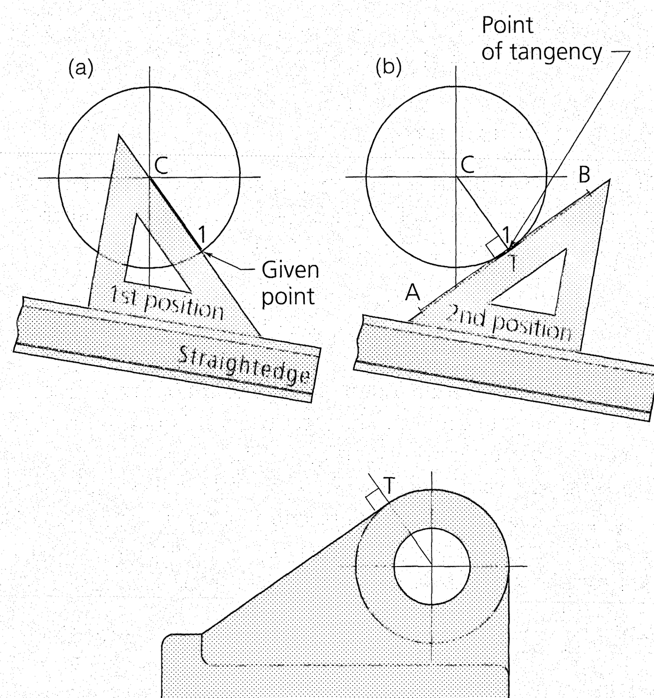


FIGURE 8.22 Drawing a Line Tangent to a Circle

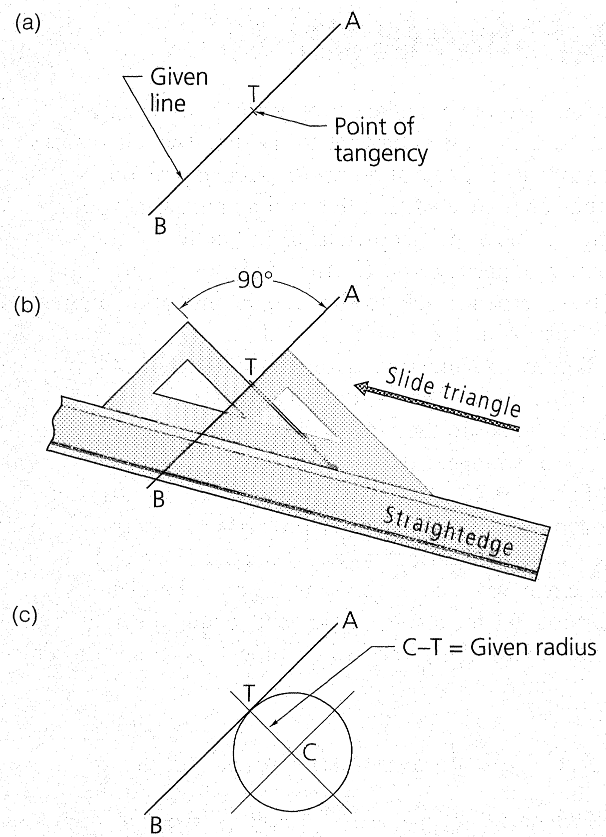


FIGURE 8.23 Drawing a Circle Tangent to a Line

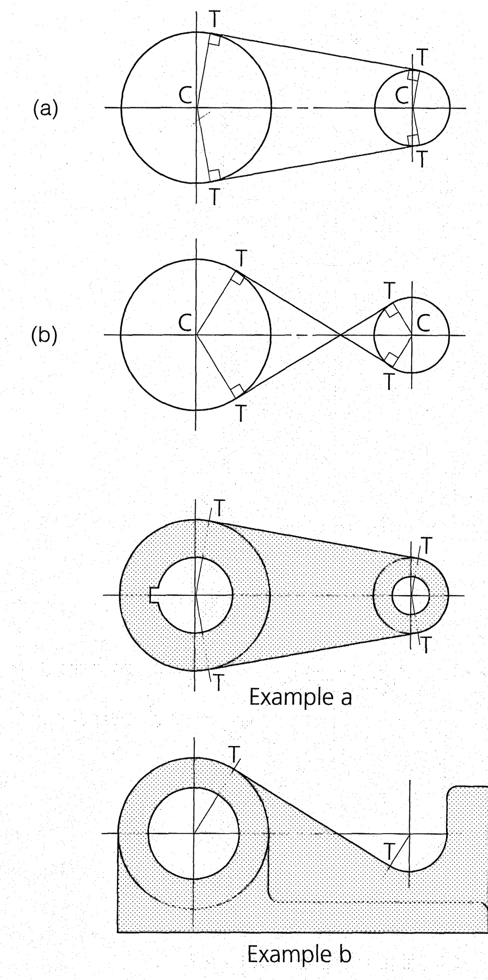


FIGURE 8.24 Tangencies of a Line and Two Circles

tangent to two circles, as shown in Figure 8.24(a) and (b). Four tangency positions are possible. In Figure 8.24(a), the lines are tangent to the outside of the circles. This is called an **open belt tangent.** In Figure 8.24(b), the lines form a **closed belt tangent.** In both examples, the circles are given. The construction is the same as for Figure 8.22.

8.5.2 Tangent Arcs

There are two methods for drawing an **arc between two perpendicular lines.** Figure 8.25(a) illustrates the construction of arc 2-3 using only a compass.

For Perpendicular Lines, with a Compass [Fig. 8.25(a)]

1. Extend the two given perpendicular lines to meet at point 1.
2. From point 1 strike a radius equal to the required radius of the tangent arc. The intersection of this radius and the given lines establishes tangent points 2 and 3.
3. Using the same radius, strike construction arcs from points 2 and 3. The intersection of these two arcs, at point C, establishes the center of the tangent arc.
4. From point C, draw arc 2-3 tangent to both perpendicular lines.
5. Locate points of tangency on both lines.

Figure 8.25(b) illustrates a second method.

For Perpendicular Lines, Without a Compass [Fig. 8.25(b)]

1. Extend the given perpendicular lines so they meet at point 1.
2. Draw a parallel line distance R from each of the given lines using the required tangent arc radius for dimension R. Point C is at the intersection of these two lines.
3. Locate tangent points 2 and 3 by extending construction lines from C perpendicular to the given lines.
4. From center point C, draw the required tangent arc from point 2 to point 3

To draw arcs that are tangent to nonperpendicular lines, use the same procedure as in Figure 8.25(b). This method is for lines at acute angles (Fig. 8.26) or at obtuse angles (Fig. 8.27). In Figures 8.26 and 8.27, the given lines have been extended to meet at point 1. Use the following steps.

For Nonperpendicular Lines [Figs. 8.26 and 8.27]

1. Draw construction lines parallel to and at distance R from the given lines using the required tangent arc radius.
2. Where these two lines intersect (point C), draw construction lines perpendicular to the given lines to establish points 2 and 3 as the points of tangency.

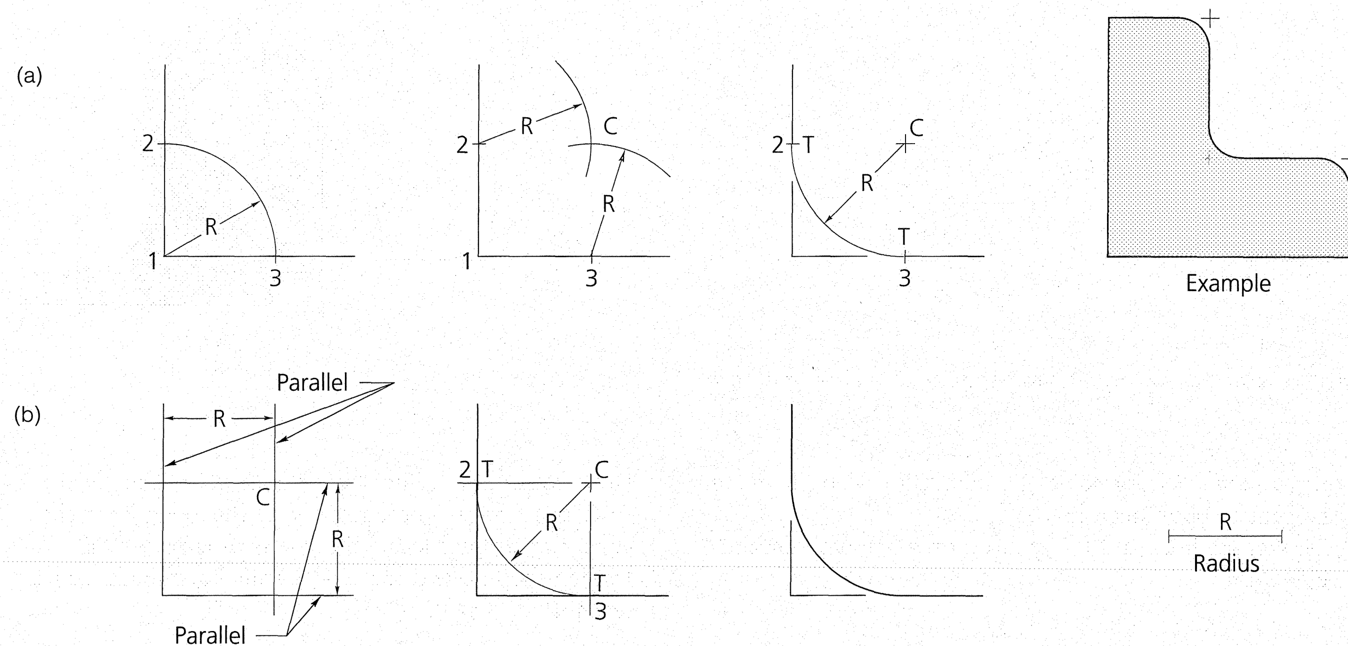


FIGURE 8.25 Drawing an Arc Tangent to Two Perpendicular Lines

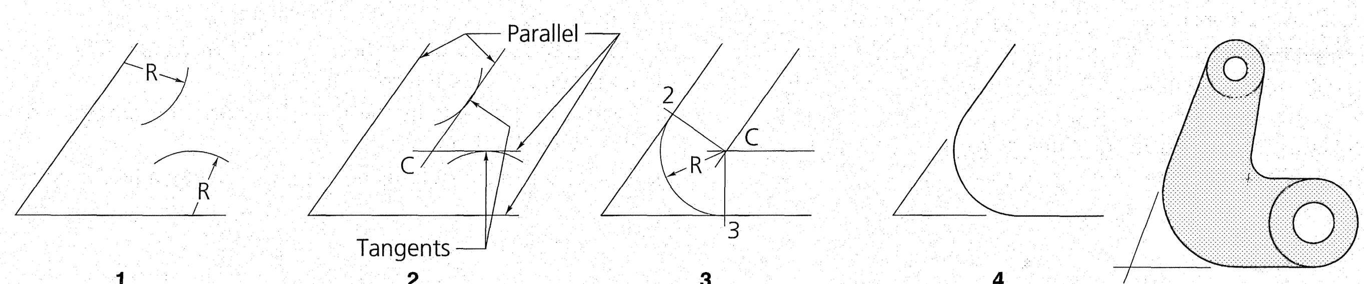


FIGURE 8.26 Drawing an Arc Tangent to Two Lines Forming an Acute Angle

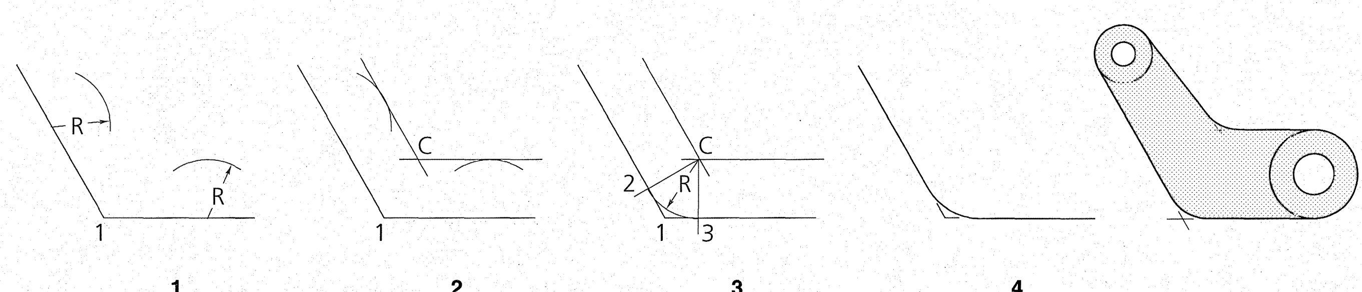


FIGURE 8.27 Drawing an Arc Tangent to Two Lines Forming an Obtuse Angle

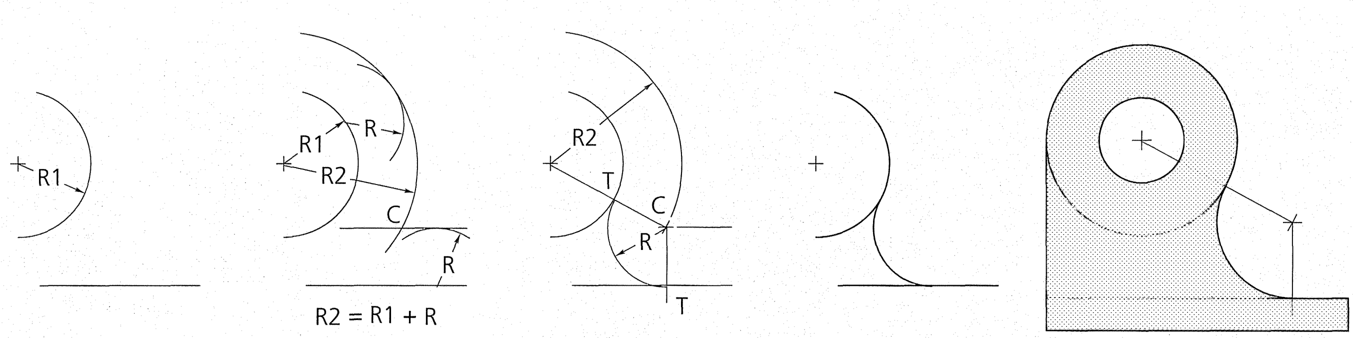


FIGURE 8.28 Drawing an Arc Tangent to a Line and an Arc

3. Draw radius R from point 2 to point 3 to establish an arc tangent to both given lines.
4. Darken the lines and the arc to form a smooth, continuous figure.

8.5.3 Drawing an Arc Tangent to a Line and an Arc

To construct an arc tangent to a line on one side and an arc on the other (Fig. 8.28), use the following procedure. The line and the arc are given along with the required radius R for the tangent arc.

1. Draw the given line and arc.
2. Draw a construction line parallel to and at distance R from the given line. Add R and R1 to establish R2. Use R2 to swing a construction arc until it intersects the construction line at point C.
3. Using R, swing an arc tangent to the line and the given arc. Use C as the center point.
4. Draw construction lines from the center of the given arc to C and from C perpendicular to the given line. These construction lines locate the points of tangency (T).
5. Darken the line and the arcs, forming a smooth, consistent line

8.5.4 Drawing an Arc Tangent to Two Arcs

To construct an arc tangent to two arcs or circles, lay out the given arcs as shown in Figure 8.29. Here R1 and R2 are given along with the distance between their centers. The radius length (R) of the tangent arc is also provided.

1. Add the radius length R to R1. Use this length to draw a construction arc.
2. Add R to R2 and draw another construction arc. The two construction arcs intersect at C.
3. Using the given radius length (R), draw the tangent arc with C as the center.
4. Locate the point of tangency (T) by drawing a line from C

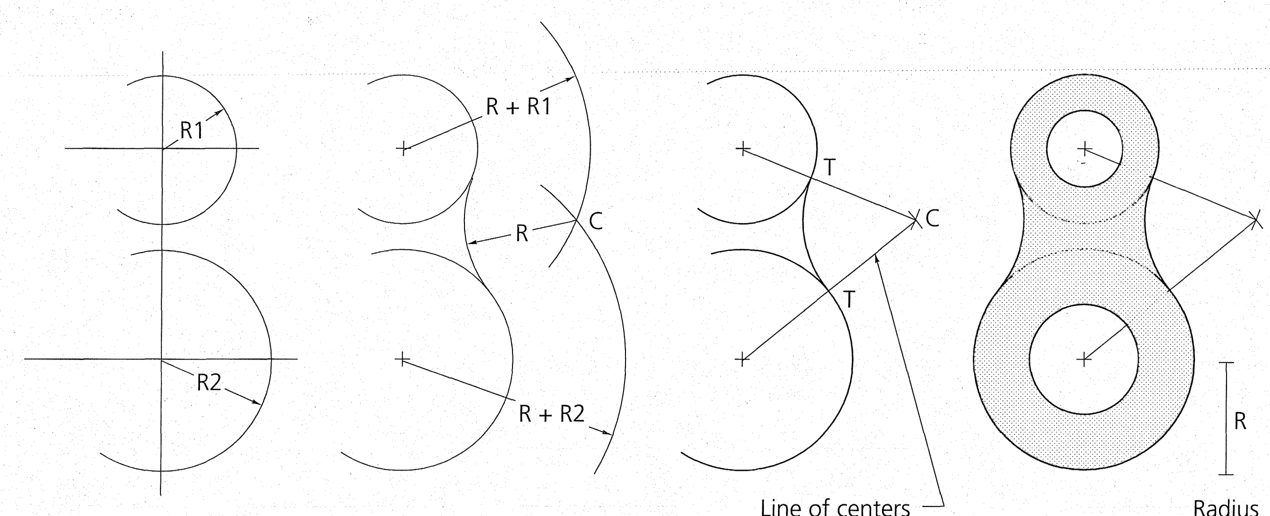


FIGURE 8.29 Drawing an Arc Tangent to Two Arcs

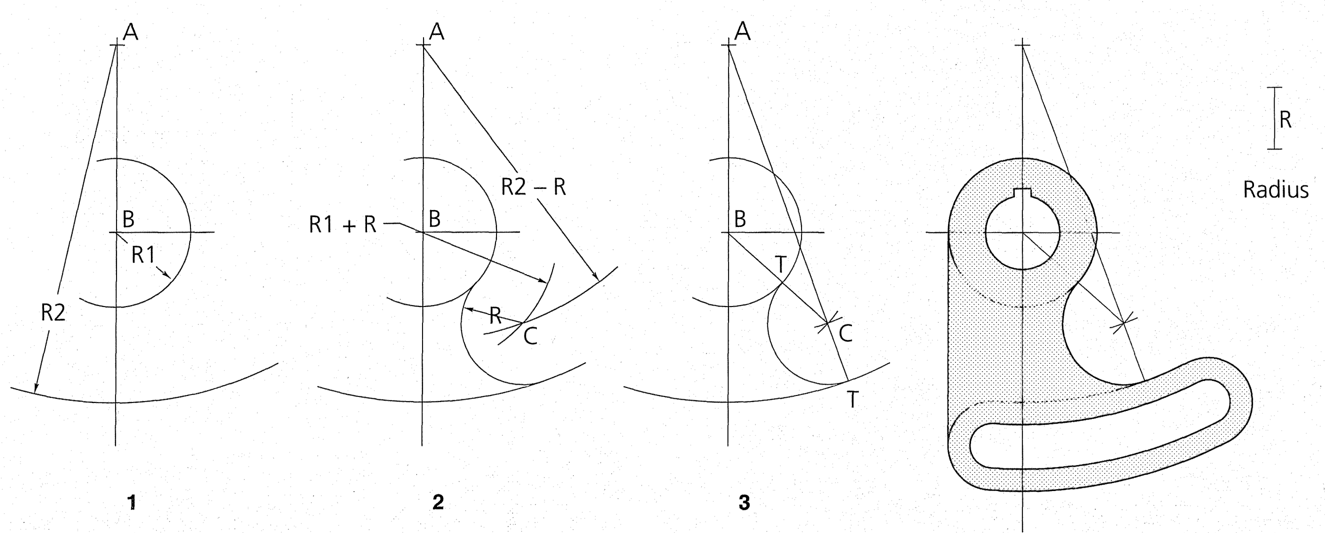


FIGURE 8.30 Constructing a Tangent Arc Between Two Arcs

to each center of the given arcs. This line is called the **line of centers.**

8.5.5 Drawing an Arc Tangent to Two Arcs with One Arc Enclosed

In Figure 8.30, a tangent arc joins two arcs. In this example, the tangent arc becomes tangent to the inside of one arc and tangent to the outside of the other. The arcs are given along with their centers, A and B. The radius length (R) of the tangent arc is also provided. The following method is used.

1. Locate centers A and B, and construct the two given arcs, Rl and R2.
2. *Add* R and R1 and use this length to draw a construction arc from B. *Subtract* R from R2, and use this length to draw a construction arc from A. The intersection of these two construction arcs locates C. Using the given tangent arc radius (R), draw an arc from C tangent to both given arcs.
3. Locate the point of tangency by drawing a line (line of centers) from A through C until it intersects the large arc at T. Locate the other point of tangency (T) by drawing a line from B to C.

8.5.6 Drawing an Arc Tangent to Two Arcs and Enclosing Both

In Figure 8.31, the tangent arc encloses both given arcs. R1, R2, A, and B are given along with the tangent arc radius R. The tangent arc is drawn by using the following steps:

1. Lay out the two given arcs as shown in the figure (R1 and R2).

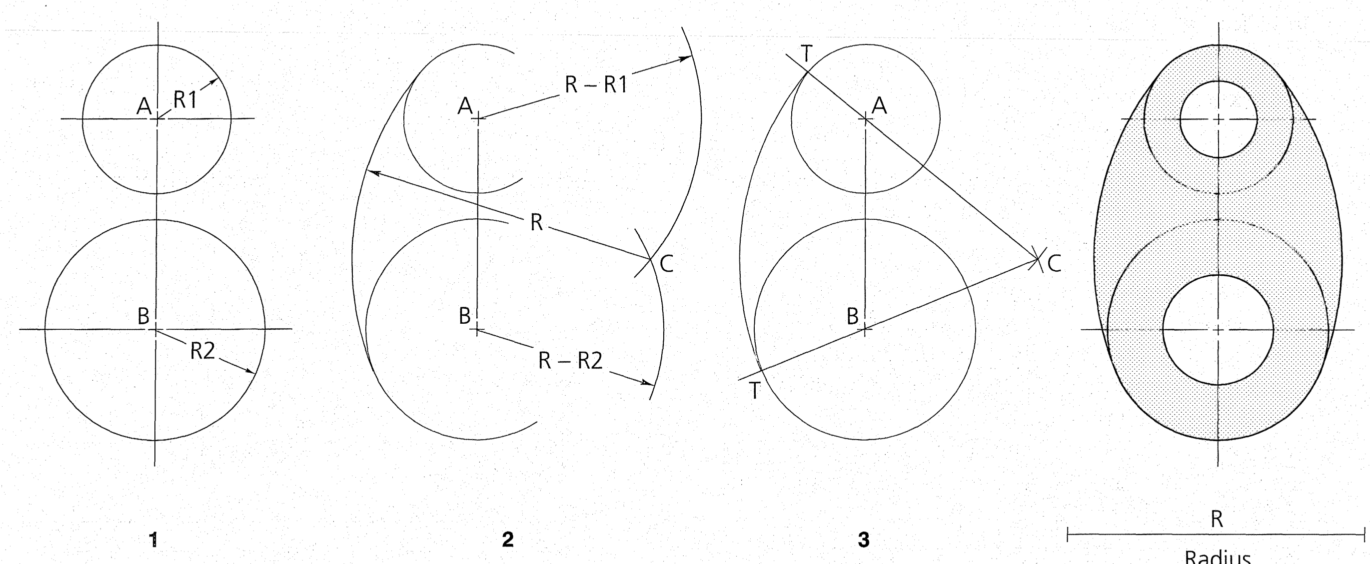


FIGURE 8.31 Drawing an Outside (Enclosing) Arc Tangent to Two Arcs

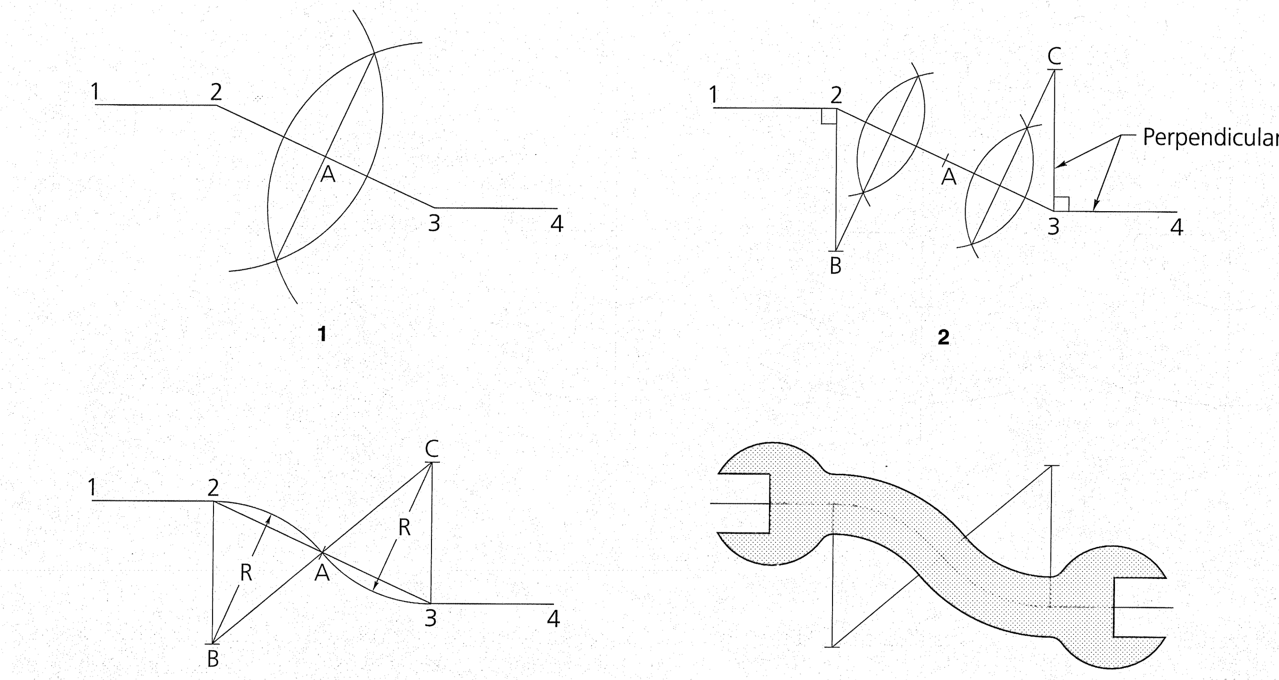


FIGURE 8.32 Drawing an Ogee Curve

2. Find C by first subtracting the radius length R1 from R. Use this length to draw a construction arc. Subtract R2 from R and draw another construction arc. The intersection of these two arcs locates C. Using R, draw an arc with C as its center and its ends tangent to the two given arcs.
3. Determine the exact point of tangency by drawing construction lines from C to A and from C to B, extending both until they intersect the arc, as shown. The intersection of these lines and the two given arcs locates the two tangent points (T).

8.5.7 Drawing Ogee Curves

An **ogee curve** is used to connect two parallel lines with tangent arcs. In Figure 8.32, lines 1-2 and 3-4, their parallel distance, and their location in space are given. The curve is constructed as follows.

1. Draw lines 1-2 and 3-4. Connect points 2 and 3 and bisect this new line (2-3) to locate point A.
2. Bisect lines 2-A and 3-A. Extend these bisectors until they intersect perpendiculars drawn from points 2 and 3. This will locate points B and C.
3. Draw arcs (R) using the distance from B to 2 (or C to 3). The points of tangency are 2 and 3 for the arcs and the lines, and A for the two arcs.

You May Complete Exercises 8.1 Through 8.4 at This Time

8.5.8 Rectifying Circles, Arcs, and Curves

Circles and arcs can be laid out (*rectified*) along a straight line. Their true length (circumference or arc length) is layed off along a straight line. All rectification is approximate but is still graphically acceptable within limits.

To **rectify** the circumference of a circle means to find the circumference graphically. In Figure 8.33, the circumference of the circle has been established by rectification.

1. Draw line 2-5 tangent to the bottom of the circle and exactly three times its diameter.
2. Draw line C-3 at an angle of 30°.
3. Draw line 3-4 perpendicular to the vertical centerline (line 1-2) of the circle.
4. Connect point 4 to point 5. Line 4-5 will be approximately equal to the circumference of the given circle.

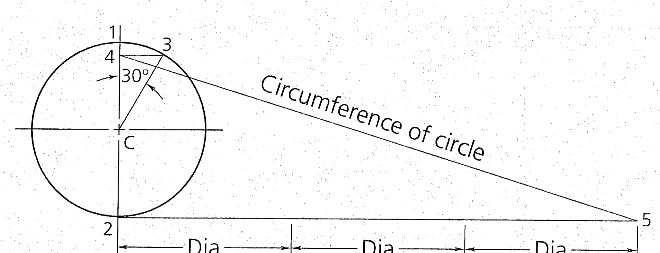


FIGURE 8.33 Rectifying a Circle

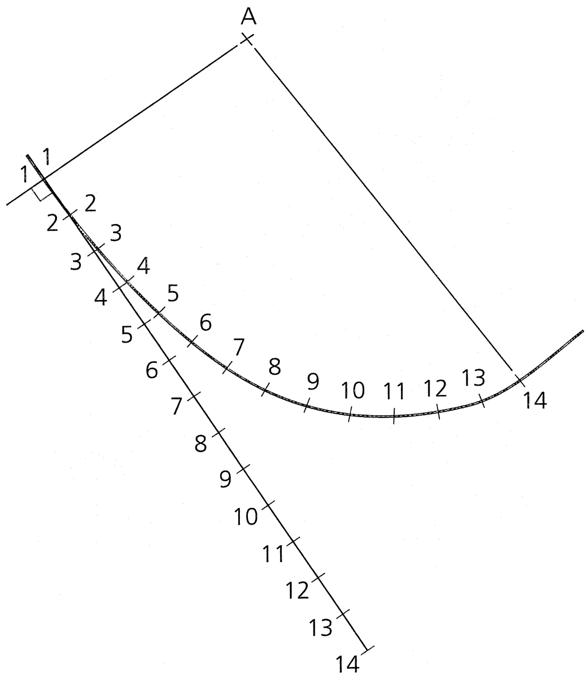


FIGURE 8.34 Rectification of an Arc

8.5.9 Approximate Rectification of an Arc

To rectify an arc or curved line, start by drawing a line tangent to one end. In Figure 8.34, the line was drawn tangent to the curved line at point 1. (Note that it is not necessary to have point A, although it does help to establish the exact tangent points.) The following steps are used.

1. Use dividers to mark off very small equal distances along the curve. The smaller the distance, the more accurate the approximation because each distance will be the chord measurement of its corresponding arc segment and, therefore, will be somewhat shorter than the arc's true length.
2. Starting at the opposite end of the arc, away from the side with the tangent line, mark off equal chords, point 14 to point 13, 13 to 12, 12 to 11, 11 to 10, and so on. Continue marking off each division until less than one full space remains, which is at point 2 in the given example.
3. Without lifting the dividers, start dividing the tangent line into the same number of segments, 2 to 3, 3 to 4, 4 to 5, and so on. The tangent line 1-14 will approximately equal the length of the given arc.

8.6 CONIC SECTIONS

A **right circular cone** is one in which the altitude and the axis coincide (the axis is perpendicular to the base). The intersection of a plane and a right circular cone is called a *conic section*. Five possible sections can result from this intersection (Fig. 8.35). The shapes formed by the sections are:

1. *Parabola* A plane (EV 1) passes parallel to a true-length element (edge) of the cone, forming the same base angle (angle between the base and the edge) and resulting in a parabola.
2. *Hyperbola* A plane (EV 2) passing through a cone, parallel to the altitude and perpendicular to the base, results in a hyperbola.
3. *Ellipse* A plane (EV 3) that cuts all the elements of the cone but is not perpendicular to the axis forms a true ellipse.
4. *Triangle* A plane that passes through the vertex and is parallel to the axis cuts an isosceles (or equilateral) triangle (front view).
5. *Circle* A plane that passes perpendicular to the axis forms a circular intersection. In Figure 8.35, a series of horizontal cutting planes have been introduced in the frontal (front) view, which project as circles in the horizontal (top) view.

8.6.1 Intersection of a Cone and a Plane

The **intersection of a cone and a plane** is established by passing a series of horizontal cutting planes through the cone (perpendicular to its axis). In Figure 8.35, the front and top views of the cone are shown along with the edge view of three planes that intersect it. To find the top view and the true shape of each intersection, use the following steps.

1. In the front view, pass a series of evenly spaced horizontal cutting planes through the cone, CP1 through CP12.
2. Each cutting plane projects as a circle in the horizontal view.
3. EV 1 intersects cutting planes 3 through 12 in the frontal view. Project intersection points to the top view. The intersection of EV 1 and the cone forms a parabola (1).
4. The true shape of the parabola is seen in a view projected parallel to EV 1. Draw the centerline of the parabola parallel to EV 1, and project the intersection points of the plane (EV 1) and each cutting plane from the front view. Distances are transferred from the horizontal view, as in dimension A.
5. Repeat steps 3 and 4 to establish the intersection of EV 2 and EV 3 with the cone. EV 2 projects as a line in the top view and as hyperbola in a true-shape view (2). EV 3 forms an ellipse in the top view and projects as a true-size ellipse in view (3).

8.6.2 Constructing an Ellipse

Two methods for constructing an **ellipse** are covered in this section: the **concentric-circle method** and the **four-center**

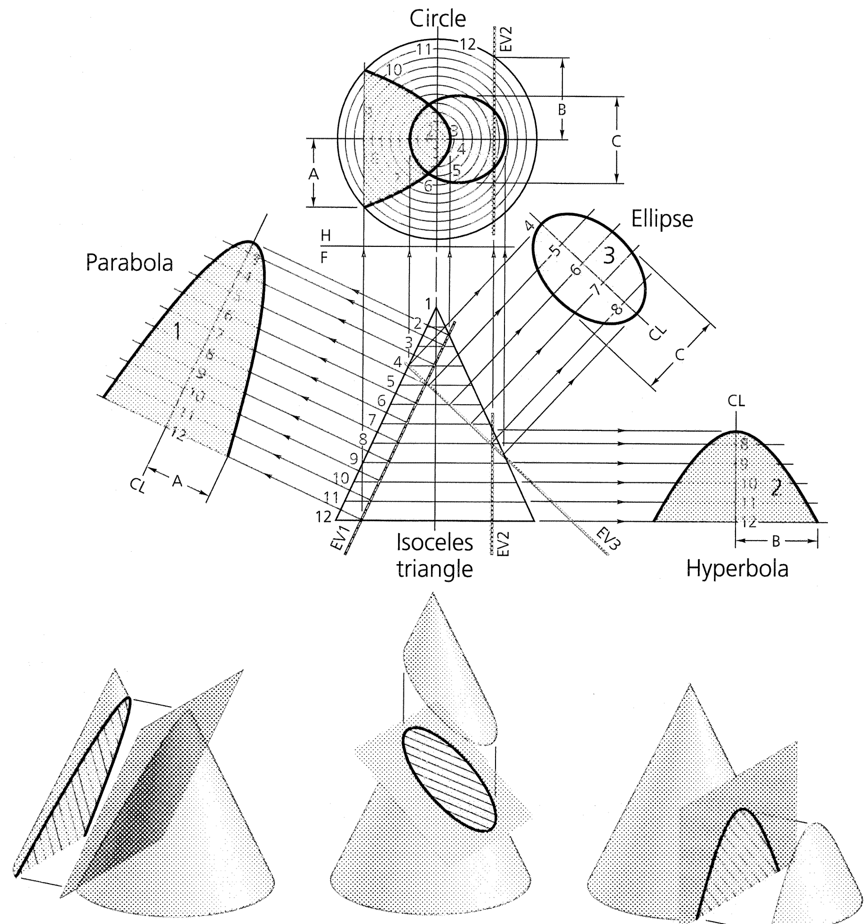


FIGURE 8.35 Conic Sections

method. Both methods are useful for constructing oddly sized or large ellipses. And both *approximate* the shape of a true ellipse. Figure 8.36 illustrates the concentric-circle method of constructing an ellipse.

Concentric-Circle Method (Fig. 8.36)

1. Given the major axis A-B and the minor axis C-D, draw concentric circles (circles of a different size with the same center point) using the axes as diameters.
2. Divide the circles into an equal number of sections. Figure 8.36 uses twelve equal divisions.
3. Where each line crosses the inner circle (point 2 or 5), draw a line parallel to the major axis; where the same line crosses the outer circle (point 1 or 4), draw a line parallel to the minor axis. The point of intersection of these two lines (point 3 or 6) will be on the ellipse.
4. Repeat this process for each division of the circles. Use an irregular curve to connect the points smoothly. It is accurate in direct proportion to the number of divisions used and points located.

Figure 8.37 shows the approximate method, also called the four-center method, for constructing an ellipse.

Approximate (Four-Center) Method (Fig. 8.37)

1. With the major axis (A-B) and the minor axis (C-D) given, connect points B and C.

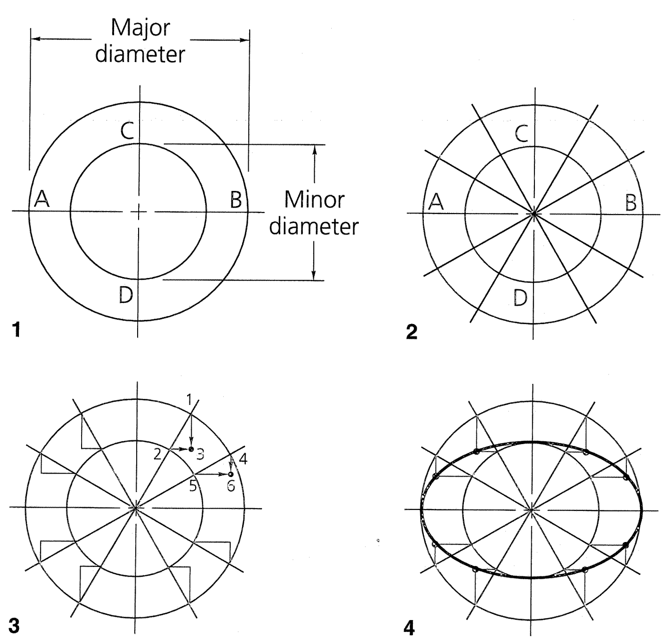


FIGURE 8.36 Drawing an Ellipse Using the Concentric-Circle Method

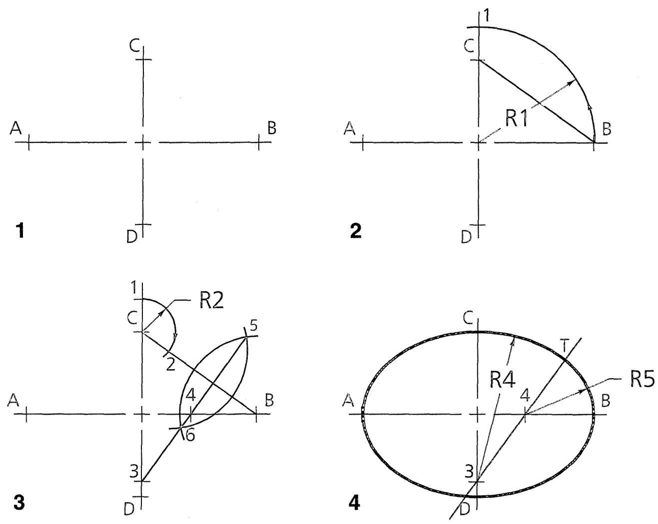


FIGURE 8.37 Drawing an Ellipse Using the Approximate Method (Four-Center Method)

2. Using the distance from the center of the ellipse to point B as the radius, strike arc R1. Point 1 is the intersection of R1 and the extended minor axis.
3. The distance from point C to point 1 establishes R2. Draw arc R2 so it intersects line B-C at point 2.
4. Bisect line B-2 and extend the bisector 5-6 so it crosses the minor axis at point 3. Point 3 is the center point for radius R4.
5. Where bisector 5-6 crosses the major axis (point 4), draw radius R5 to establish the sides of the ellipse at point A and point B. R4 is the radius for the upper and lower arcs at points C and D of the ellipse.

These two methods work best when *the minor axis is at least 75% of the major axis.* When the minor axis is too small in comparison to the major axis, the top and bottom of the ellipse are flattened. The closer the major axis and the minor axis are in length, the more accurate the ellipse.

8.6.3 Constructing a Parabola Using a Rectangle or Parallelogram

A **parabola** is the result of an intersection between a cone and a plane passed parallel to one of its elements. It is a plane curve, generated by a point moving so that its distance from a fixed point, known as the *focus,* is always equal to its distance from a fixed line. Parabolas are used in the design of surfaces that need to reflect sound or light in a specific manner. The construction of a parabola is demonstrated with a rectangle in Figure 8.38(a) and with a parallelogram in Figure 8.38(b).

1. Divide side BC into an even number of equal parts and side AB into half as many equal parts.
2. Connect the points along AB and CD to E.
3. Draw parallel lines from the points along BC to where

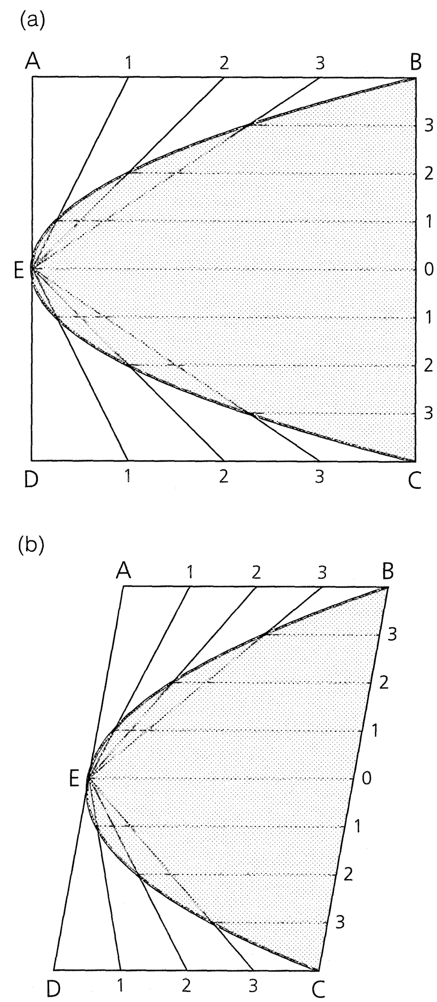


FIGURE 8.38 Drawing a Parabola

they intersect the lines drawn in step 2. The intersection points are points along the parabola's curve.
4. Connect the intersection points using an irregular curve. The greater the number of divisions, the greater the accuracy of the curve.

8.6.4 Constructing a Parabola by Establishing the Intersection of a Plane and a Cone

Figure 8.39 illustrates the step-by-step procedure for constructing a parabola by establishing the intersection of a plane and a cone.

1. Draw the given cone and the intersecting plane in the front and top views. A parabola is formed by an intersecting plane that is parallel to one of the cone's elements-edge lines (front view).
2. Draw any number of concentric circles in the top view. The greater the number of circles, the greater the accuracy of the parabola. [Figure 8.39(b) uses only two circles in order to provide a clearer picture of the process.]

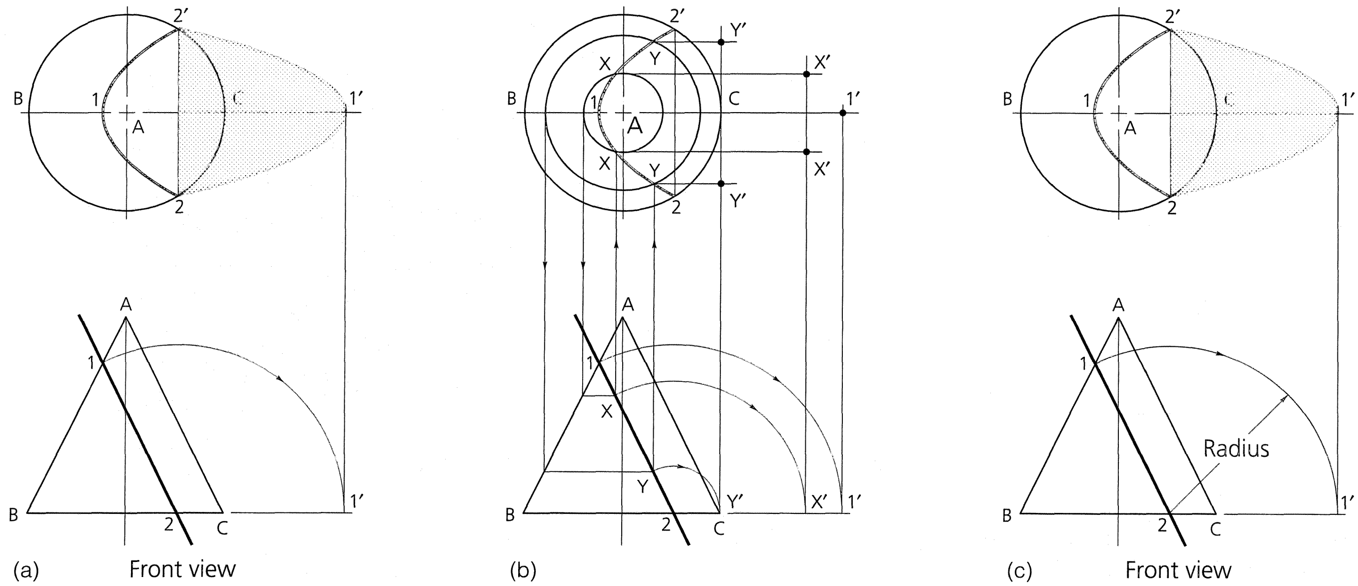


FIGURE 8.39 Constructing a Parabola

3. Project the circle to the front view. The intersection of the circle (seen as an edge) and the plane establishes points X and Y in the front view
4. Project points X and Y to the top view and complete the top view of the parabola.
5. Using point 2 as the center, draw arcs using lengths 2-Y, 2-X, and 2-1 until they intersect the base plane in the front view. Project these points to the top view.
6. Draw horizontal lines from each intersecting point in the top view until they intersect with corresponding points projected from the front view.
7. Connect these points to form the true view of the parabola.

8.6.5 Connecting Two Points with a Parabolic Curve

Figure 8.40 shows three parabolic curves. In each case, points X, Y, and 0 are given. The following steps are used for the construction.

1. Draw lines X-0 and Y-0.
2. Divide each line into the same number of equal parts, and number the divisions.
3. Connect the corresponding points with construction lines.
4. Sketch a smooth curve that is tangent to each of the elements, as shown.
5. Use an irregular curve to draw the curve.

8.6.6 Constructing a Hyperbola

A **hyperbola** is a plane surface (curve) that is formed by the intersection of a right circular cone and a vertical plane.

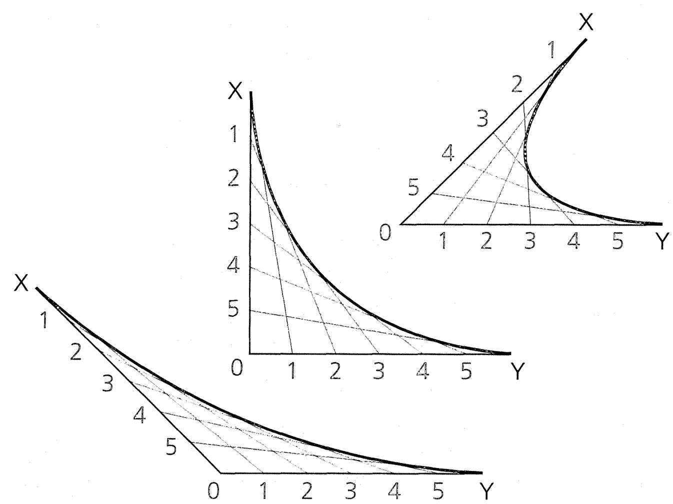


FIGURE 8.40 Drawing Parabolic Curves

Figure 8.41 illustrates the procedure for drawing a hyperbola:

1. Draw the cone and the plane in the top and front views [Fig. 8.41(a)].
2. Construct a number of planes parallel to the base [Fig. 8.41(b)]. These planes form concentric circles when they intersect the cone (top view). The greater the number of planes, the greater the accuracy of the hyperbola.
3. In the front view, the intersection of each edge of the planes intersects the vertical plane and establishes four points in the top view: X, X′, Y, and Y′.
4. The vertical plane appears as an edge in the top view.

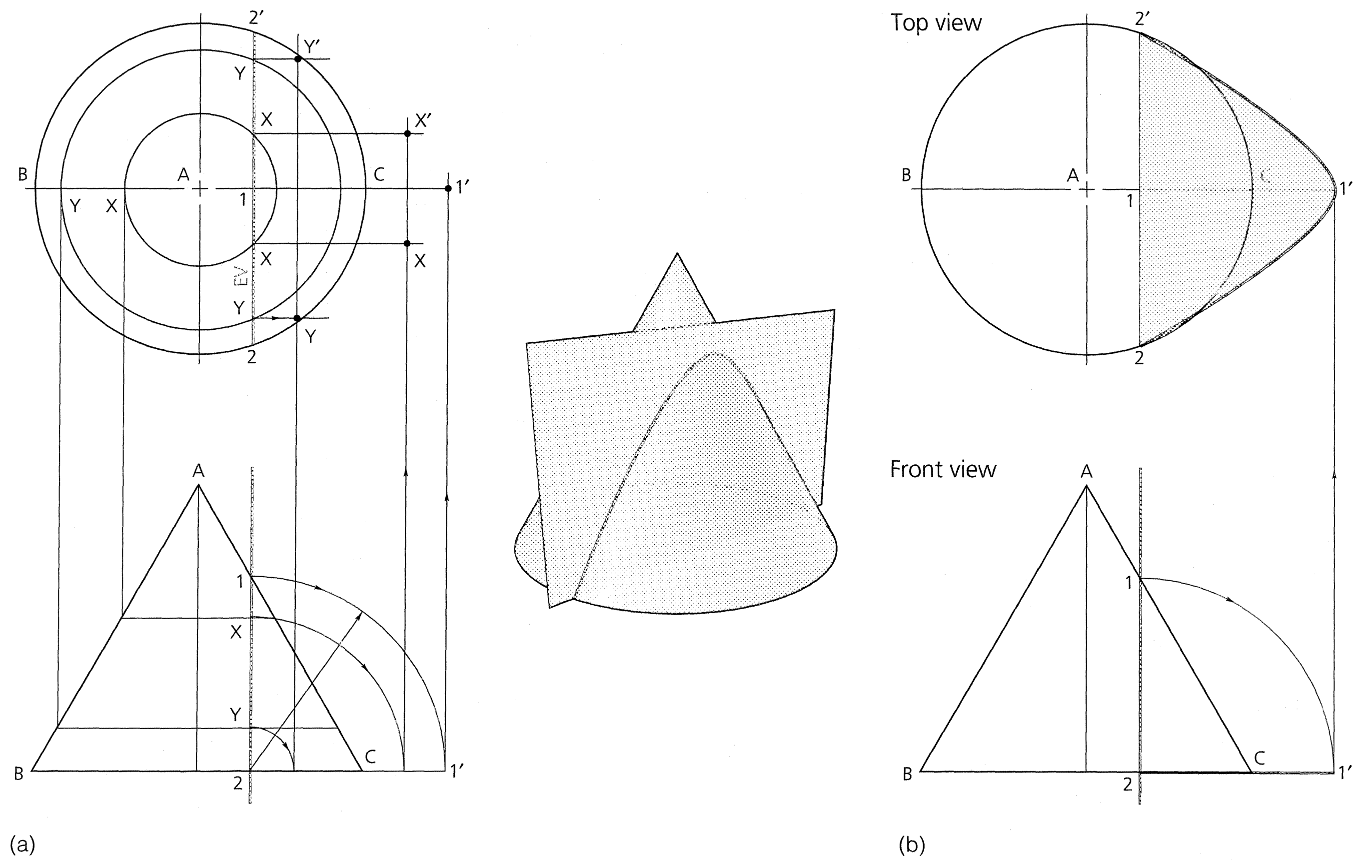


FIGURE 8.41 Constructing a Hyperbola

Draw horizontal construction lines from each intersecting point as shown in Figure 8.41(b).

5. Draw the required arcs in the front view until they intersect the base plane of the cone. Project these points to the top view.
6. The intersection of corresponding points establishes points on the hyperbola in the top view.
7. Use an irregular curve to draw the curve.

8.6.7 Drawing a Spiral of Archimedes

A **spiral of Archimedes** is a plane curve generated by a point moving away from or toward a fixed point at a constant rate while a radial line from the fixed point rotates at a constant speed. Figure 8.42 shows a spiral of Archimedes. To draw one, use the following steps.

1. Draw centerlines with a center point at 0, as shown.
2. Establish an equal number of angles; twelve angles of 30° each were used in the example.
3. Divide any line into the same number of equal divisions; in the example, line 0-12 is used (divisions A through L).
4. Draw the construction arcs from each point to the corresponding angle.
5. Each intersection of an arc and an angle establishes one point of the spiral.
6. Use an irregular curve to connect the points.

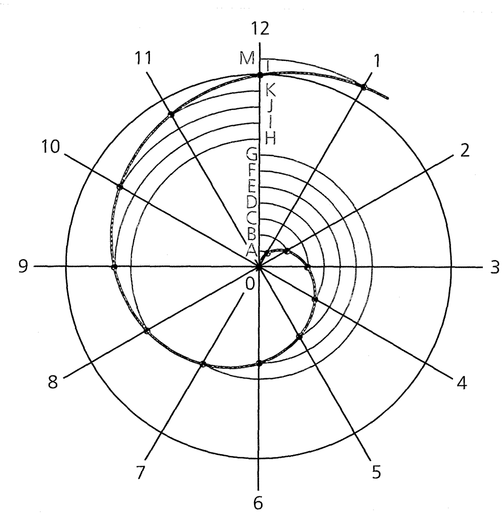


FIGURE 8.42 Drawing a Spiral of Archimedes

8.6.8 Constructing an Involute of a Line, a Triangle, a Square, or a Circle

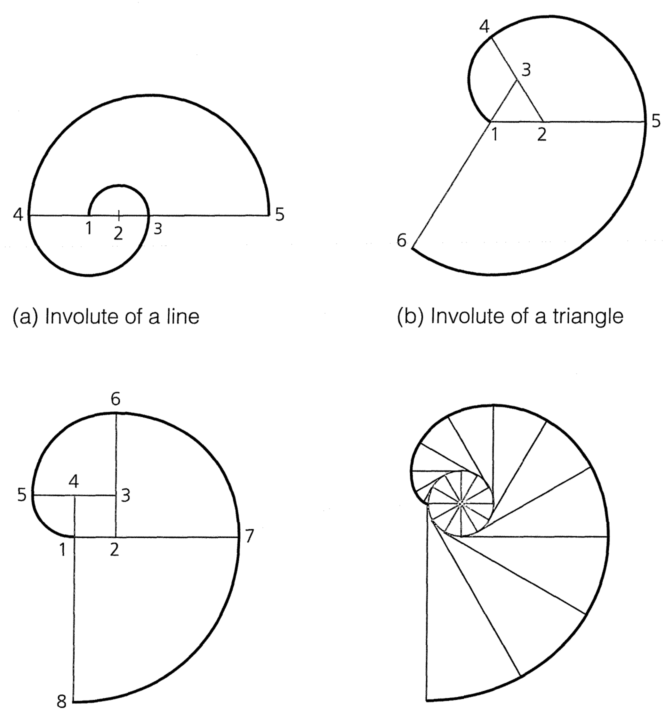
An **involute** is a plane curve traced by a point on a thread kept taut as it is unwound from another curve. Figure 8.43 shows four kinds of involutes.

In Figure 8.43(a), the **involute of a line** is constructed by first drawing the given line 1-2. Point 2 becomes the center of the first arc (radius 1-2), point 1 the center of the second arc (radius 1-3), and point 3 the center of the third arc (radius 3-4).

The **involute of a triangle** is constructed by drawing the triangle [Fig. 8.43(b)]. Use point 3 as the center of the first arc (radius 3-1). Extend lines 2-3, 1-2, and 3-1. Use point 2 as the center of the second arc (radius 2-4) and point 1 as the center of the next arc (radius 1-5).

The **involute of a square** is constructed by drawing the square and extending each side line [Fig. 8.43(c)]. Point 4 becomes the center of the first arc (radius 4-1), point 3 the center of the second arc (radius 3-5), point 2 the center of the third arc (radius 2-6), and point 1 the center of the fourth arc (radius 1-7).

The **involute of a circle** is constructed by drawing the circle and dividing it into equal angles [Fig. 8.43(d)]. Draw tangent construction lines from the end of each angle. Mark off along each tangent the length of each circular arc. Use an irregular curve to connect the endpoints of the arc and tangent lines with a smooth curve.



(a) Involute of a line (b) Involute of a triangle

(c) Involute of a square (d) Involute of a circle

FIGURE 8.43 Drawing Involutes

8.7 HELICES

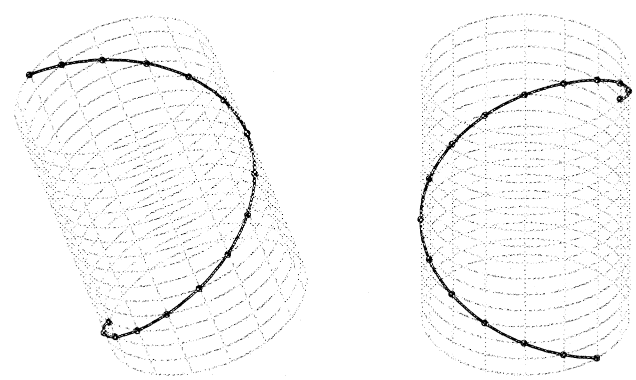
A **cylindrical helix** is a double-curved line drawn by tracing the movement of a point as it revolves about the axis of a cylinder. Figure 8.44 shows two revolved positions of a cylindrical helix modeled in 3D. The resulting curve is traced on the cylinder by the revolution of a point crossing its right sections at a constant oblique angle. The point must travel about the cylinder at a uniform linear and angular rate. The linear distance (parallel to the axis) traveled in one complete turn is called the **lead.** This type of helix is called a cylindrical helix. A variety of industrial products are based on the cylindrical helix, including fasteners and springs. The stairway in Figure 8.1(b) was designed with a cylindrical helix.

If the point moves about a line that intersects the axis, it is a **conical helix.** The generating point's distance from the axis line changes at a uniform rate. A helix can be either *right-handed* or *left-handed.*

8.7.1 Constructing a Helix

The techniques for constructing a cylindrical helix and a conical helix use the same steps. Start the construction by radially dividing the end view (curve) into an equal number of parts (Fig. 8.45). The lead is divided into the same number of parts. Use the following steps.

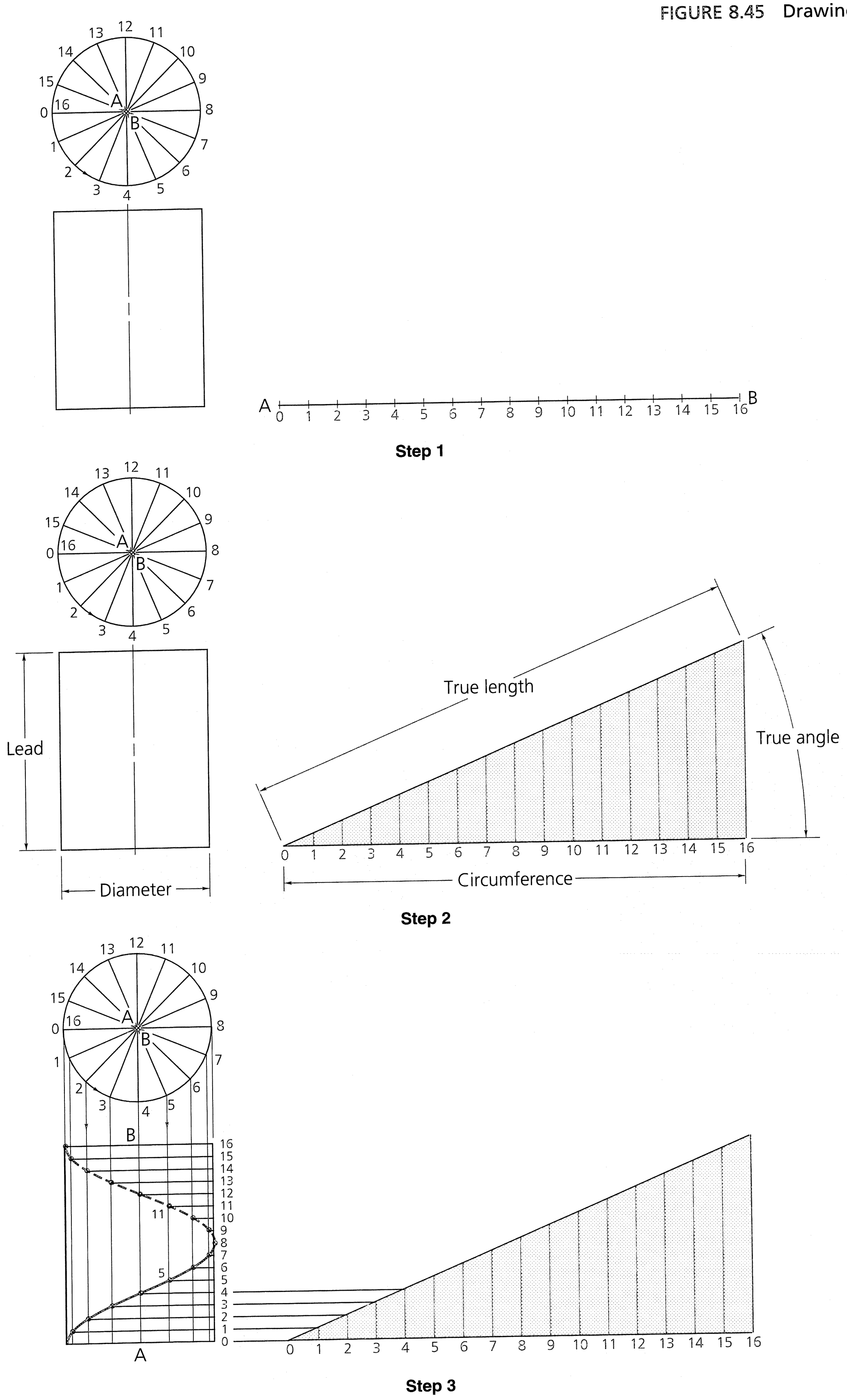
1. Draw the right-handed cylindrical helix by first dividing the circular end view into equal divisions. Also divide the lead into equal parts (sixteen were used in the example).
2. Label the points on both views.
3. Project the end view divisions to the front view as vertical elements on the surface of the cylinder. In the front view, establish a series of points on the surface of the cylinder. Each point represents a position of the generating point as it rotates about the axis.
4. You can develop the cylindrical helix by unrolling the cylinder's surface. The helix line is a straight line on the

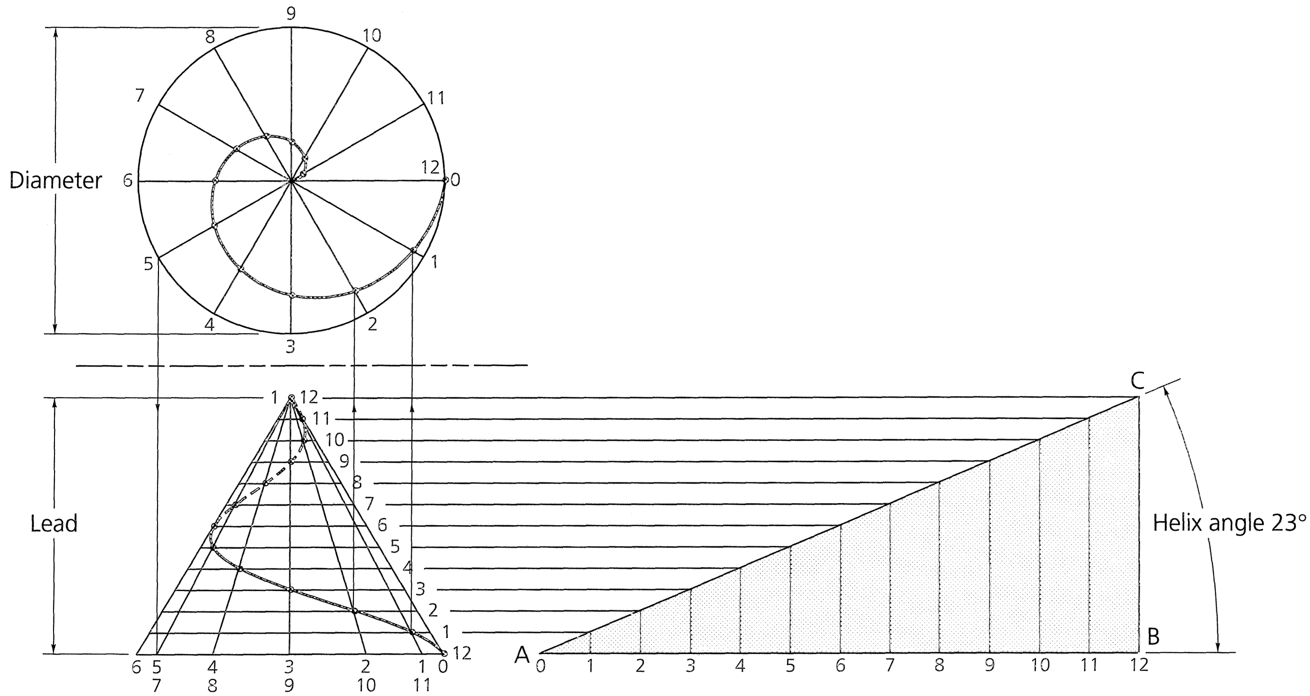


(a) (b)

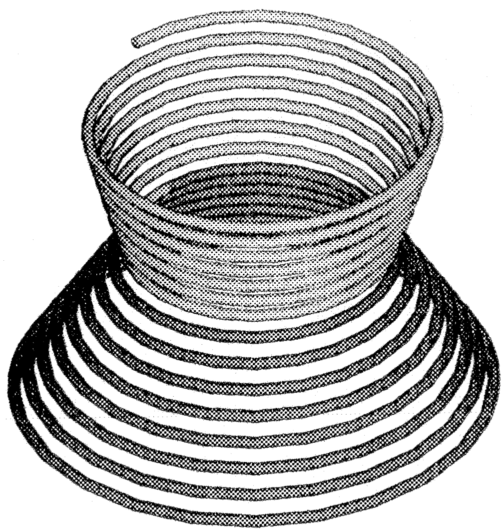
FIGURE 8.44 3D Model of a Cylindrical Helix

FIGURE 8.45 Drawing a Cylindrical Helix





(a) Drawing a conical helix



(b) Parametric model of helical spring design

FIGURE 8.46 **Conical Helices**

development. The angle the helix line makes with the baseline is called the **helix angle** (true angle).

To construct a conical helix, you must know its *taper angle* (angle between the cone's axis and an element on the cone surface) and lead. In Figure 8.46(a), the lead and the circle divisions are established as for a cylindrical helix. Elements determined in the top (end) view appear in the front view as straight lines, intersecting the vertex of the cone. Lead elements are drawn as horizontal lines. Points on the surface of the cone are located at the intersection of related elements. Figure 8.46(b) is an example of a helical design created with a parametric modeling system

You May Complete Exercises 8.5 Through 8.8 at This Time

8.8 GEOMETRIC CONSTRUCTION USING CAD

In CAD, anything placed on a drawing is a geometry **item,** or **entity.** Basic geometric entities include points, lines, arcs, and circles (Figs. 8.47 and 8.48). Entities are used to create

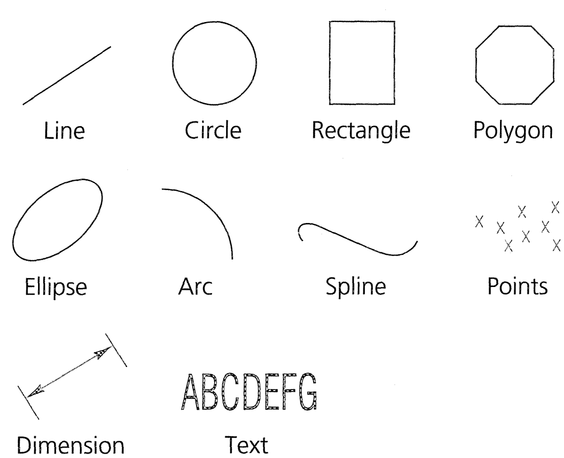


FIGURE 8.47 **Geometric Entities**

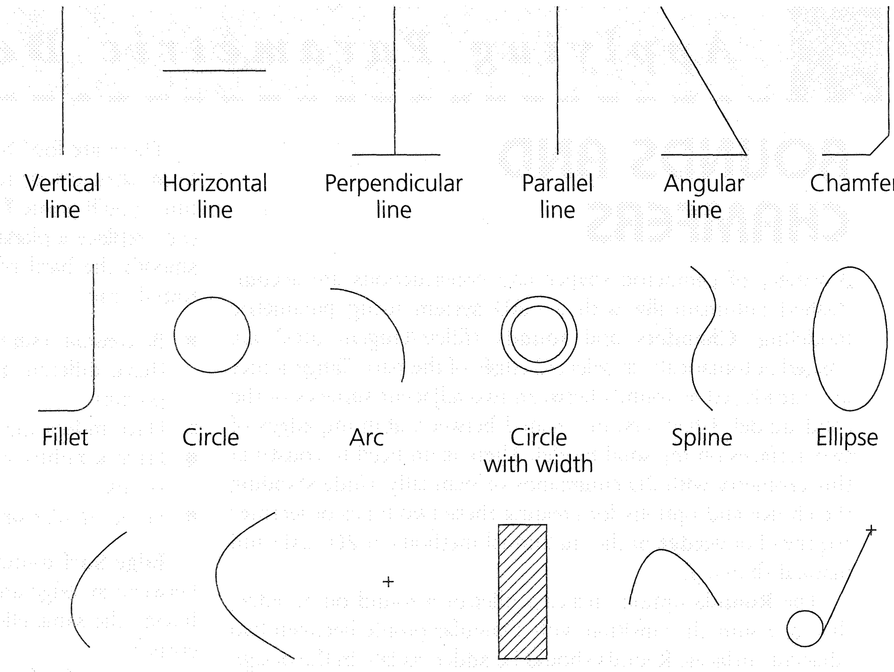


FIGURE 8.48 Additional Geometric Entities

any type of geometric feature. These CAD-based constructions can be substituted for many of the geometric constructions introduced in the first portion of this chapter. Of course, you still need to understand fundamental construction techniques to complete drawings on CAD systems. The techniques described for CAD systems are similar for all systems, but differ in the command names used. The commands presented here are for AutoCAD.

8.8.1 Location of Geometry

There are three ways to define a location on a drawing in CAD: (1) by digitizing the location (**free digitizing**), (2) by entering the location's coordinates via the computer keyboard or menu (**explicit entry**), or (3) by snapping to a location on an entity using a **reference,** which AutoCAD calls object snaps (**OSNAPS**). **OSNAPS** can specify a location on existing geometry. For example, the endpoint, center, or midpoint of an entity and the intersection of two entities (Fig. 8.49) are object snaps.

8.8.2 Basic Construction

Regardless of whether a 2D or 3D system is used, creating the geometry is the first step in drawing a *part* with a CAD system. The basic construction instructions are used to create geometry entities such as lines, arcs, and circles. Entities are the building blocks of every part. To construct geometry, the system must know the location, the size, and the appearance of the entity. To specify an entity's size, a value is given and the explicit coordinates are either typed or defined by freehand digitizing.

OSNAP options (Fig. 8.49) enable the creation of geometry relative to an existing entity; either the **ENDPOINT**, **MIDPOINT**, or **CENTER** of an entity can be referenced. The intersection of two entities can also be referenced with an **OSNAP** option (**INT**ersection). Lines have two **OSNAPS**, **END**points and a **MID**point. Circles and arcs have at least one **END**point and a **CEN**ter.

A CAD program assists your drawing efforts by finding many points automatically. For example, a typical problem in manual drafting and geometric construction is to find the

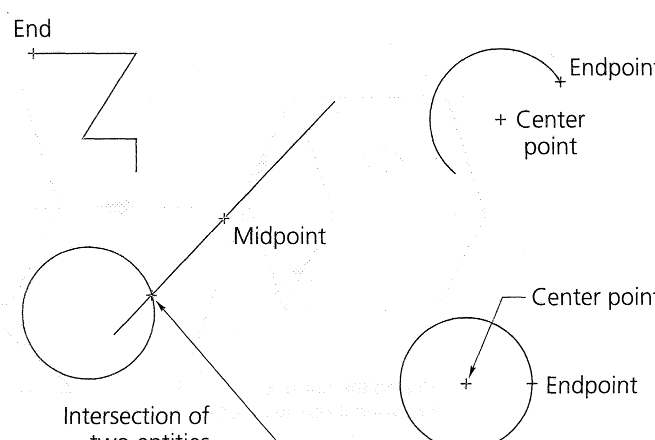


FIGURE 8.49 Geometry Terminology

Applying Parametric Design . . .

ROUNDS AND CHAMFERS

A variety of geometric shapes and constructions are accomplished automatically with a CAD system using parametric modeling. **Chamfers** and **rounds** (fillets/tangent arcs) are created automatically at selected edges of the part. Tangent arcs are introduced as rounds between two adjacent surfaces of the solid model. Chamfers are created between abutting edges of two surfaces on the solid model. There is no need to construct this geometry with 2D commands or manually. Understanding the choice and options for creating these two types of features requires knowledge of the traditional methods in 2D CAD and manual drawing.

The **Rounds** option creates a fillet or a round on an edge, that is, a smooth transition with a circular profile between two adjacent surfaces. Rounds should be added as late in the design as possible. There are cases where rounds should be added early; but in general, wait until later to add the rounds. You might also choose to place all rounds on a layer and suppress that layer to speed up your working session.

There are four basic types of rounds to consider, *edge, edge to surface, remove surface,* and *surface to surface*. Much of the time, you'll create **Edge** rounds. These do not remove surfaces (i.e., replace a planar surface with a rounded surface) but only smooth the hard edges between two adjacent surfaces. These rounds can:

- Be *constant* (single radius for all selected edges) or *variable* (have different radii specified at edge vertices *and* datum points)
- Have radius values of zero
- Have a radius value *determined* by a datum point or edge vertex
- Be of circular or conic cross section

Edge-Surf rounds (see Fig. A) create a transitional surface between an edge and a selected surface (tangent to the latter), having the same effect as rolling a ball bearing along an inside corner.

Remove Surf rounds (see Fig. B) can completely replace a surface with a rounded surface. The radius is automatically determined from the selected edges and surfaces adjacent to the one being replaced.

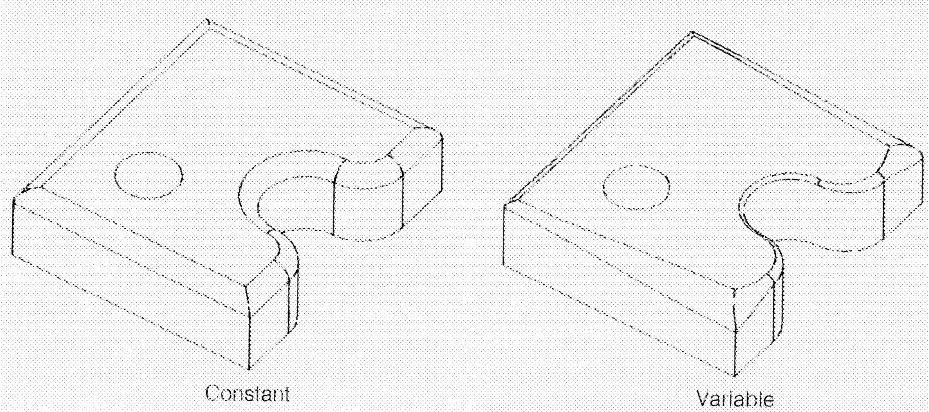


FIGURE A "Edge Round" Examples

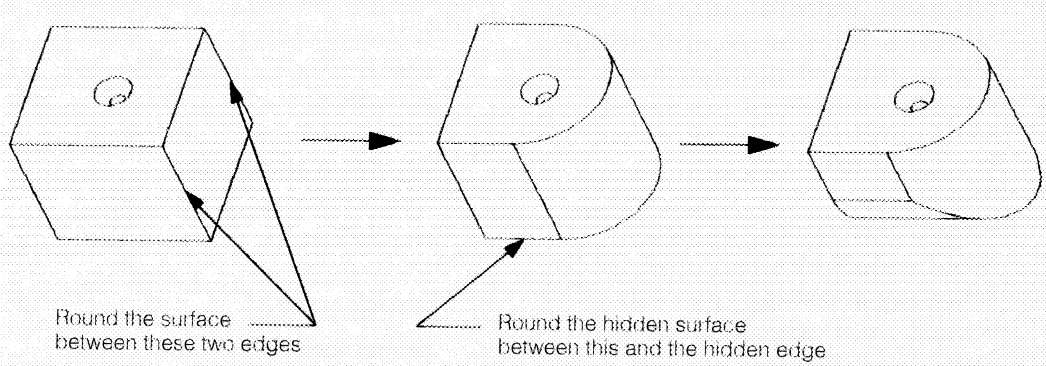


FIGURE B "Remove Surface Round" Examples

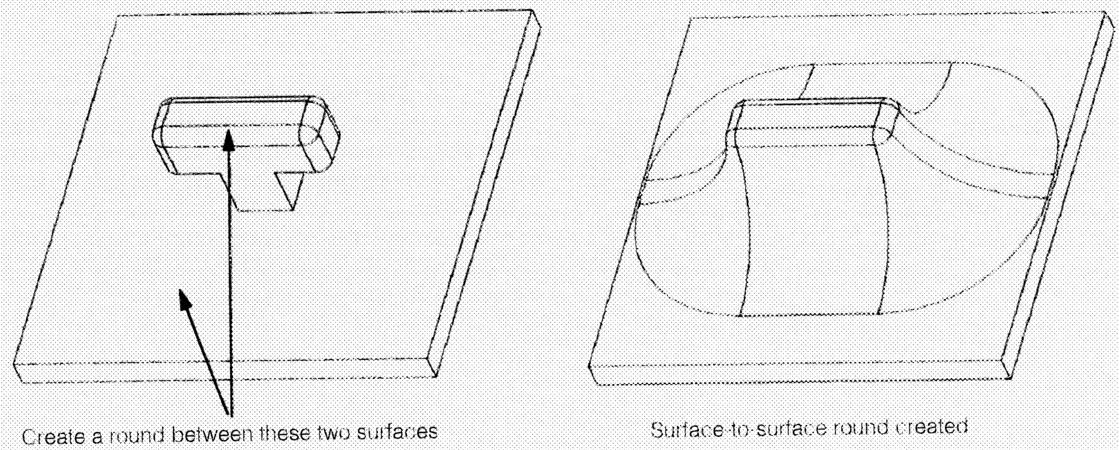


FIGURE C **Example of a Surface-to-Surface Round**

Surf-Surf (surface-to-surface) rounds (see Fig. C) are used to form transitional surfaces across multiple surfaces that don't necessarily share a common edge. They can add and remove material at the same time. These rounds are always of constant radius.

Use the **Edge** command specifically when creating rounds that do not completely remove adjacent surfaces.

A constant-radius round uses a single radius for all selected edges.

To Create a Constant-Radius Round

1. Choose **Round** form the SOLID menu.
2. Choose **Edge** and **Constant**, and **Circular** or **Conic**, and then Done.
3. Pick the edges to be rounded. Use any combination of the EDGE SELECT menu commands.
4. Enter the radius of the round using the RADIUS VALUE menu options.

The angle part provided here (see Fig. D) has two edges that require a round. The **Round** command is given, and the two edges are selected with the resulting round (see Fig. E). One round is on the outside edge of the part (a round) and the other provides a rounded internal feature (a fillet). The values of the rounds could be the same or different and can be modified at any stage of the design. Note that the rounds are children of the two surfaces that they connect.

An **edge chamfer** removes a flat section of material from a

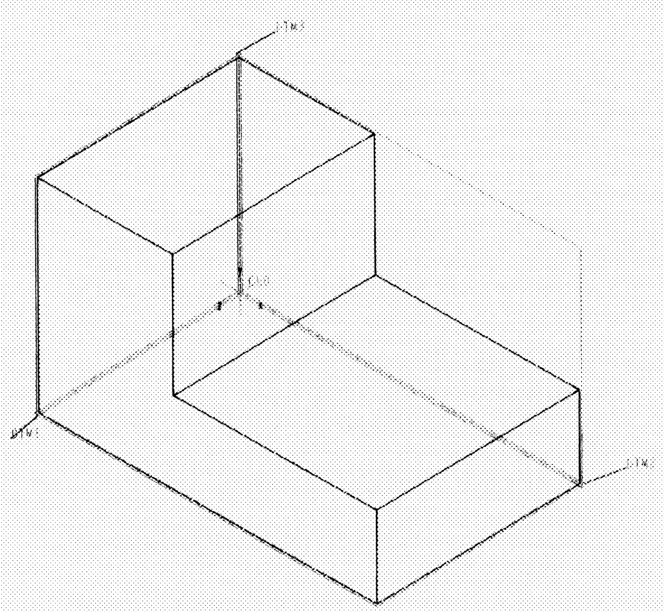


FIGURE D **Angle Part**

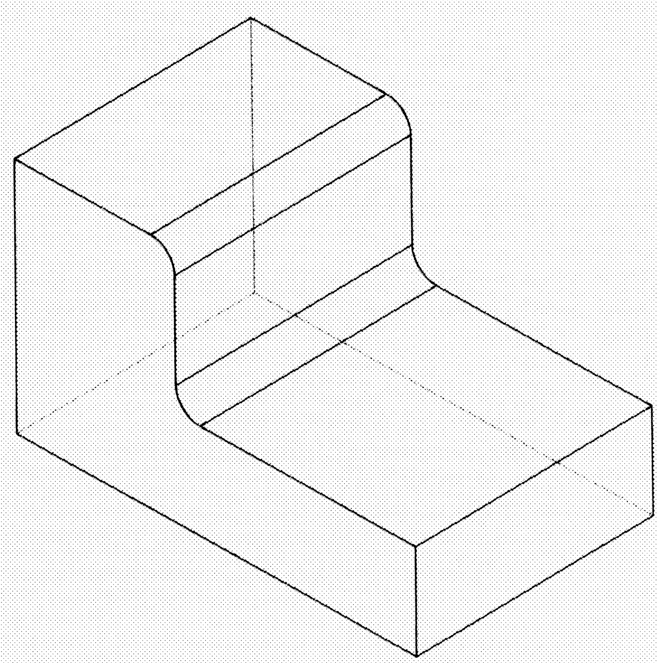


FIGURE E **Adding Fillets and Rounds with the Round Command**

Continues

selected edge to create a beveled surface between the two original surfaces common to that edge. Multiple edges may be selected. There are four dimensioning schemes for edge chamfers (see Fig. F).

- **45 × d** This dimensioning scheme creates a chamfer that is at an angle of 45° to both surfaces and a distance d from the edge along each surface. The distance is the only dimension to appear when modified. 45 × d chamfers can only be created on an edge formed by the intersection of two perpendicular surfaces.
- **d × d** This dimensioning scheme creates a chamfer that is a distance d from the edge along each surface. The distance is the only dimension to appear when modified.
- **d1 × d2** This dimensioning scheme creates a chamfer at a distance d1 from the selected edge along one surface and a distance d2 from the selected edge along the other surface. Both distances appear along their respective surfaces when modified.
- **Ang × d** This dimensioning scheme creates a chamfer at a distance d from the selected edge along one adjacent surface at a specified angle to that surface.

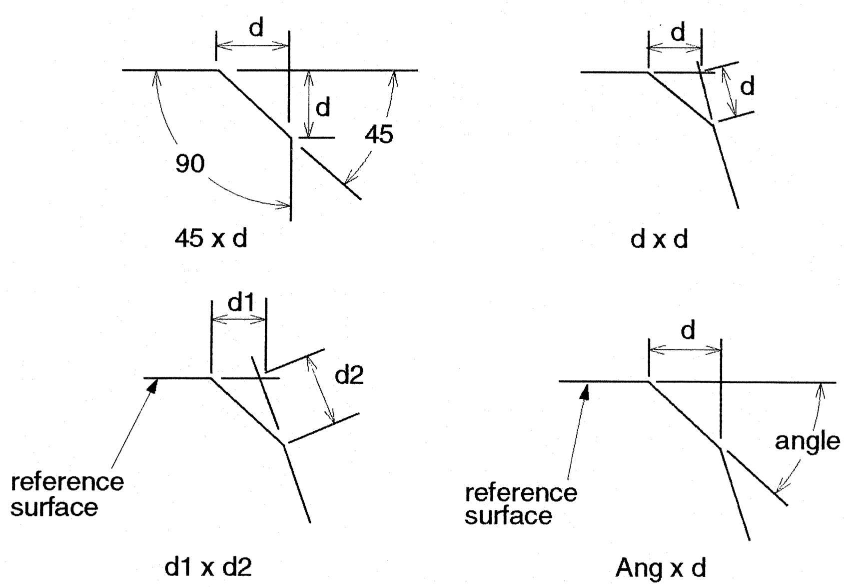


FIGURE F Chamfer Dimensioning Schemes

These schemes appear as options under the CHAMFER menu, which appears as soon as you choose **Edge** from the CHAMFER menu.

To Create a 45 × d and d × d Edge Chamfer

1. Choose **Chamfer** from the SOLID menu.
2. Choose **Edge** from the CHAMFER menu.
3. Choose the **45 × d** or **d × d**.
4. Enter the chamfer dimension.
5. Select the edges to chamfer. Remember that for a **45 × d** edge chamfer, the surfaces bounding an edge must be at 90° to each other.

To Create a d1 × d2 Chamfer

1. Choose **Chamfer** from the SOLID menu.
2. Choose **Edge** from the CHAMFER menu.
3. Choose the **d1 × d2.**
4. Input a distance along a surface to be selected.
5. Input a second distance.
6. Pick the surface along which the first distance will be measured, and pick the edge to chamfer.

To Create an Ang × d Chamfer

1. Choose **Chamfer** from the SOLID menu.
2. Choose **Edge** from the CHAMFER menu.
3. Choose the **Ang × d.**
4. Input an angle from a surface to be selected.
5. Input distance.
6. Select the surface from which the values will be measured, and specify the reference.
7. Pick the edge to chamfer and the appropriate dimensioning references.

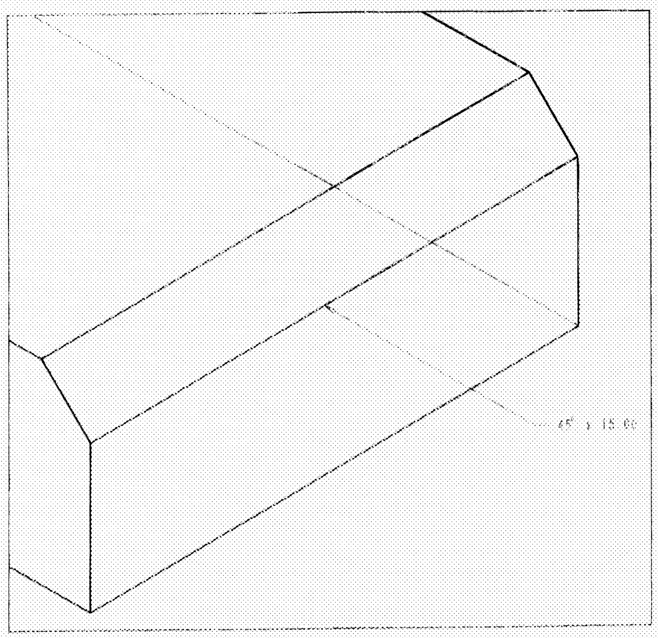


FIGURE G **Adding a 45 × 15 Chamfer Feature to the Front Edge of the Part**

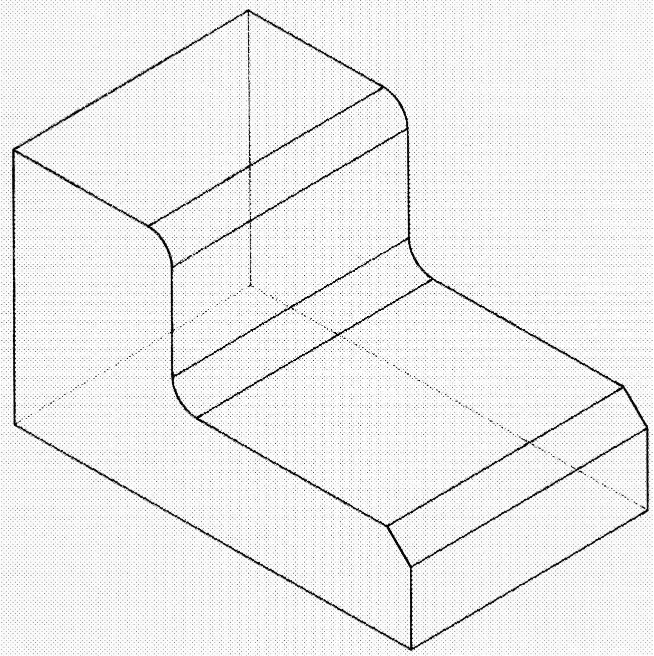


FIGURE H **Part with Chamfer and Rounds**

The **45 × d** option (see Fig. G) was used to chamfer the front edge of the angle part (see Fig. H). The next feature added to the part is a slot. The slot is created, dimensioned, and regenerated in the **sketcher** (see Fig. I). The slot is created thru the part (see Fig. J). A 45 × 5 chamfer is added to the upper edge of the slot (see Fig. K). The chamfer is then modified to 45 × 10 (see Fig. L). The finished part (see Fig. M) has rounds, a chamfer, and a chamfered slot.

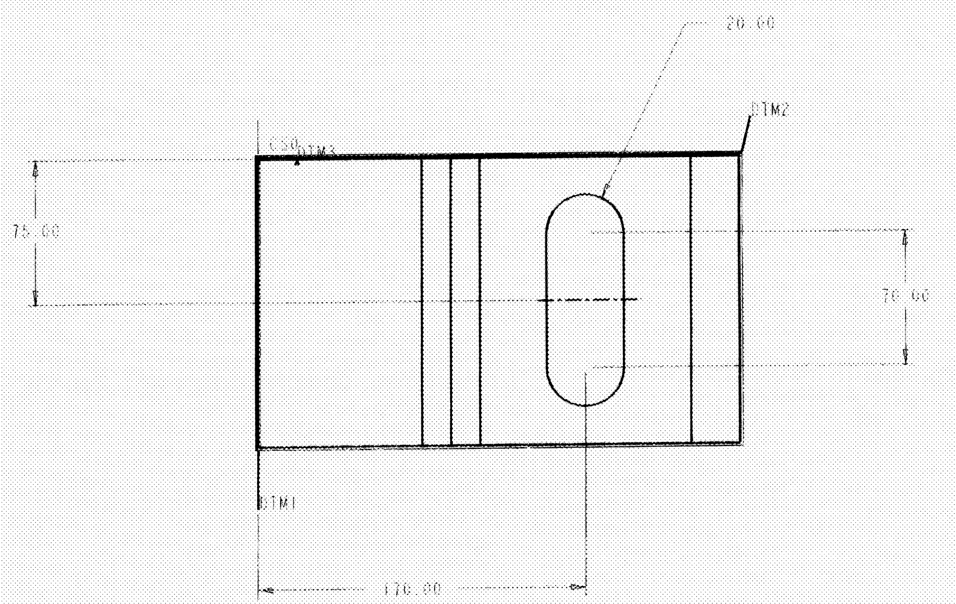


FIGURE I **Creating the Slot in the Sketcher**

Continues

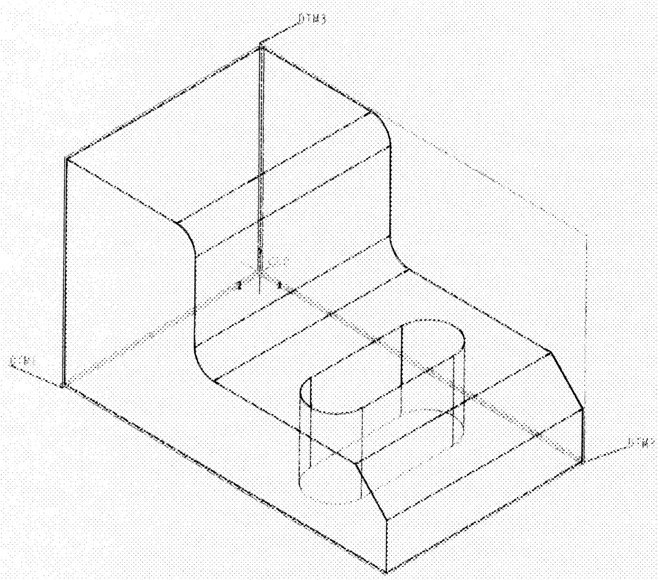


FIGURE J Part with Slot

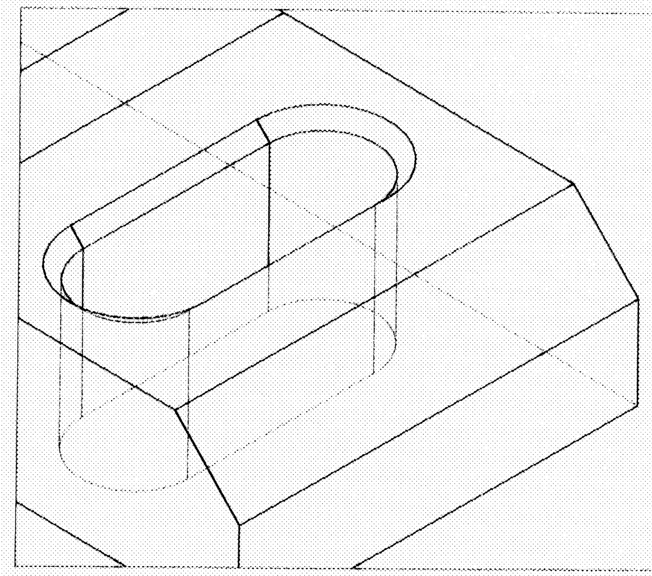


FIGURE K 45 × 5 Chamfer Added to Slot Edge

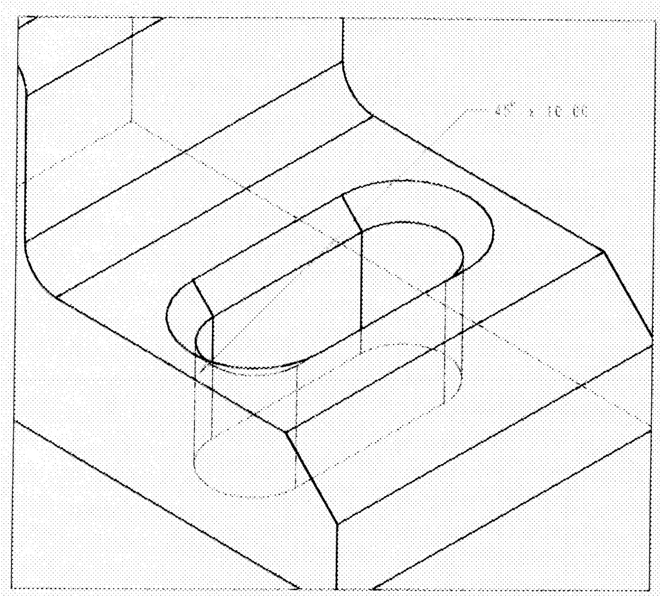


FIGURE L Chamfer Modified to 45 × 10

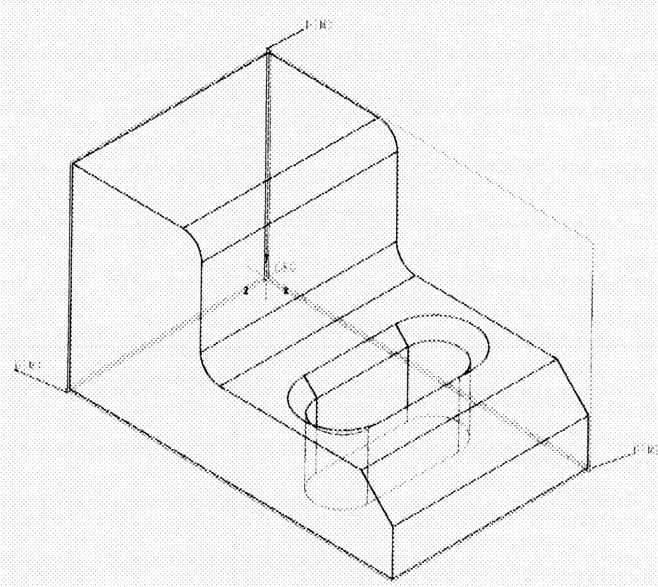


FIGURE M Completed Part with Rounds, Slot, and Chamfers

(Continued)

center of a circle or an arc. The **POINT** command using the **CEN**ter **OSNAP** reference will automatically put a point at the center of a circle.

In the following discussion, any commands or options to commands enclosed by brackets < > are the **defaults** for that command.

8.9 GEOMETRY ENTITIES

In CAD, the term **geometry entity** includes geometric forms, groups, figures, text, labels, dimensions, and cross-hatching. As an engineer or designer, you will specify the shape, size, color, and location of entities. Figures 8.47 and 8.48 show a range of geometry entities available on CAD systems.

8.9.1 Points

As discussed in the first part of this chapter, points are the simplest entities; they serve as references or as placement coordinates for other entities. You can use a digitizer or the keyboard to enter coordinate values and the **POINT** command to place a point at the specified location. Each point has an **X** and a **Y** coordinate (and a **Z** coordinate in 3D).

In Figure 8.50, three points have been drawn by free digitizing: **D1, D2,** and **D3.** Free digitizing is the same as *picking* a position on the tablet or with a mouse. The "BEFORE" illustration [Fig. 8.50(a)] indicates the digitized locations with an **x** at each location. In the "AFTER" illustration [Figure 8.50(b)], the three points are indicated by crosses (+). A variety of point styles are available; here the cross was previously selected using AutoCAD's **SETVAR** option for **PDMODE** set to 2. The following AutoCAD command was used:

Command: **POINT** (give the **POINT** command by typing or from the screen menu)
Point: **D1, D2, D3** (pick three positions; use <RETURN> between picks to recenter the **POINT** command)
Command: **REDRAW** (repaints the screen and shows the points as a cross)

8.9.2 Lines

Lines are entities that connect two endpoints of a line segment. Each point has **X** and **Y** coordinates (and a **Z** coordinate if you are using a 3D CAD system). Endpoints may be specified explicitly or referenced from existing geometry.

You can use the **LINE** command to create a line, a series of connected lines, or several separate lines. When connected lines are created, the endpoint of one line is also the start point of the next line. Lines that can be created include:

- A series of connected lines
- A closed region with connected lines (using **CLOSE** at end of command)
- A horizontal or a vertical line (using **ORTHO**)
- A line at an angle to an existing line
- A line parallel or perpendicular to an existing line
- A line tangent to a circle, a line, or a point

In Figure 8.51 a line was created by free digitizing. A series of connected lines was created in Figure 8.52 using the same command, except that **D3** and **D4** were picked instead of hitting <RETURN> at the second **To Point** prompt. The following command was used in Figure 8.52:

Command: **LINE**
From Point: **D1** (pick the starting position)
To Point: **D2** (pick the ending position of the line)
To Point: <RETURN> (ends the command)

Drawing a Horizontal or Vertical Line. You can create a horizontal or a vertical line by first turning on the orthogonal (**ORTHO**) option. This option limits your movement to the **X** and **Y** axes from your last location. Figure 8.53 shows a horizontal line created using the **LINE** command with **ORTHO** turned on. The first digitize (pick) established the starting point of the line, and the second digitize established the direction and length.

Drawing a vertical line is similar to creating a horizontal line, except that the line is aligned with the **Y** axis instead of the **X** axis. In Figure 8.54, the **LINE** command was selected and two points were digitized to create a vertical line. The first point established the line's starting point, and the second point established the line's direction and distance.

(a) Before (b) After

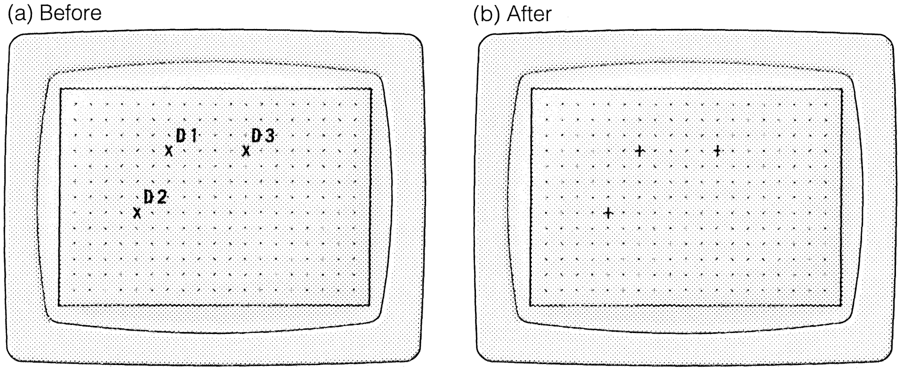


FIGURE 8.50 Drawing Points

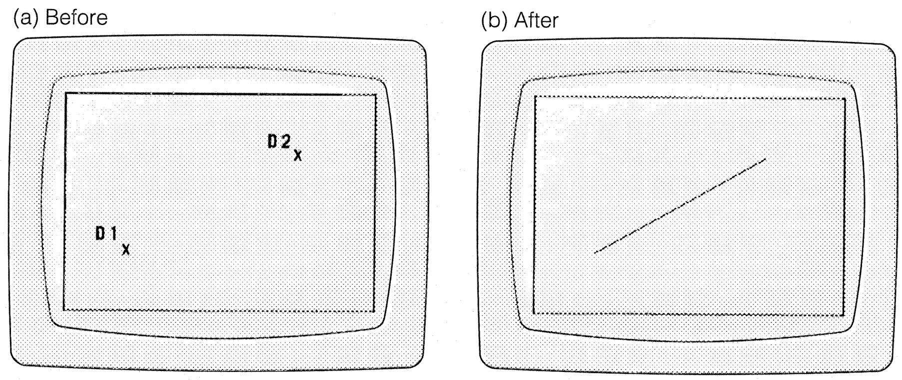


FIGURE 8.51 Drawing a Line

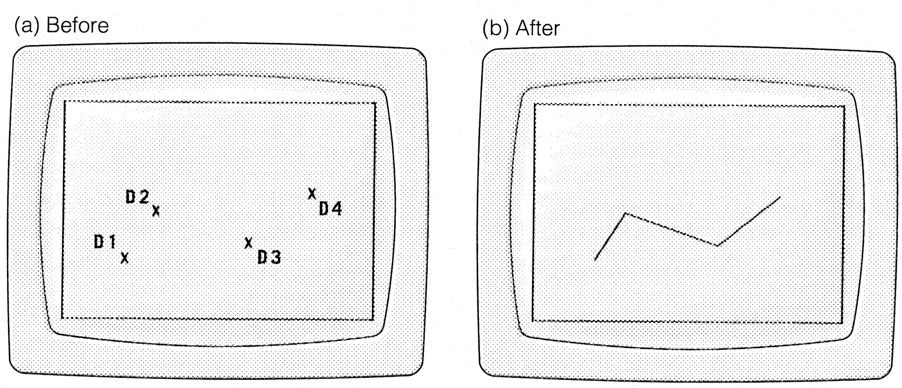


FIGURE 8.52 Drawing Multiple Lines

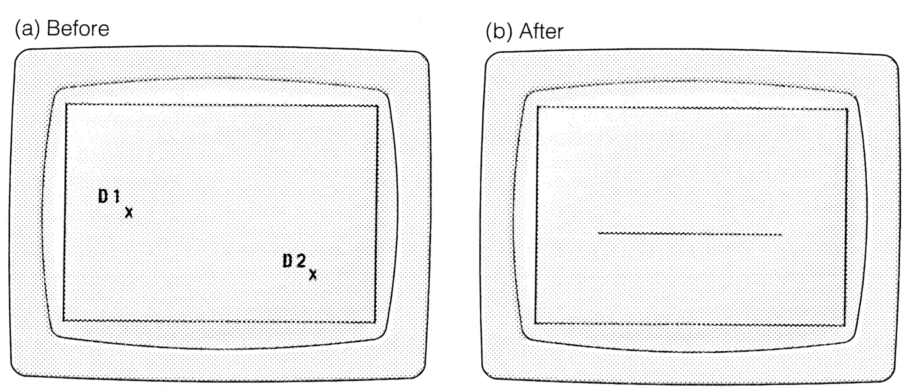


FIGURE 8.53 Drawing a Horizontal Line

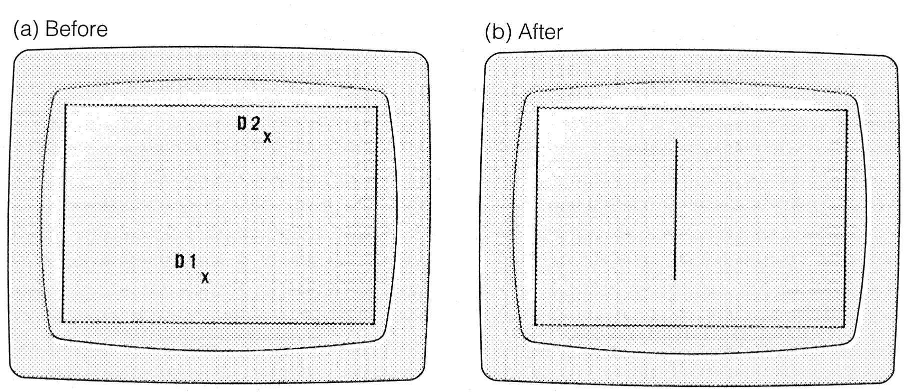


FIGURE 8.54 Drawing a Vertical Line

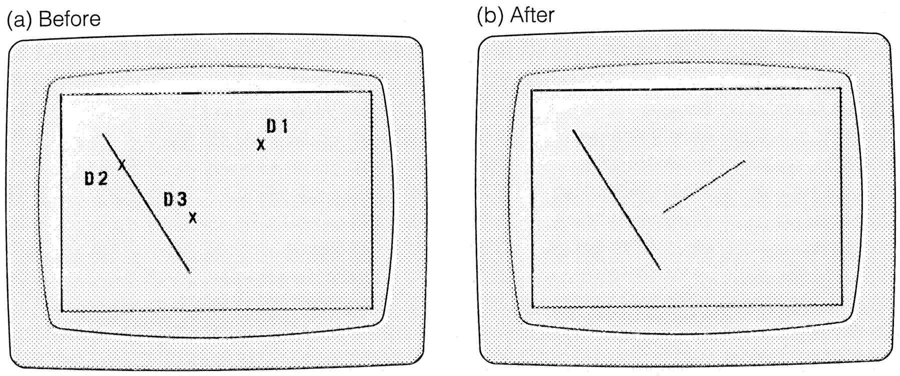


FIGURE 8.55 Drawing a Line Perpendicular to an Existing Line

Drawing a Perpendicular Line. You can draw a line perpendicular to an existing line using the **LINE** command in conjunction with the perpendicular (**PER**) **OSNAP**. Figure 8.55 shows a line that was created perpendicular to an existing line. The **LINE** command was selected as follows:

> `Command:` LINE
> `From point:` D1 (digitize the location of the line's starting point)
> `To point: per` (pick the OSNAP PER option)
> of D2 (pick the line)
> `To point:` D3 (digitize the location of the endpoint of the line)

Drawing a Tangent Line. To create a line tangent to an arc or a circle, use the **OSNAP** tangent option (**TAN**). In Figure 8.56, the **LINE** command was selected. Point **D1** was the starting point (at the end of the existing line), and **D2** identified the circle to which the line is to be tangent. Because two tangency positions are possible for each circle, the digitized points must be near the point of tangency (the side you wish the line to connect to the circle). The following command was used:

> `Command:` LINE
> `From point:` END (select the OSNAP END)
> of D1 (pick near the end of the line)
> `To point:` TAN (select the OSNAP TAN)
> of D2 (select the circle on the side of the desired tangency)

8.9.3 Circles

Using a CAD system, a circle has its start point and endpoint at the 3 o'clock position and its origin at the center. You can create circles in several different ways, depending on the type of information available. You can give three digitized circumference points, the radius or diameter value, or digitize the radius or diameter.

Drawing Circles by Specifying Three Points on the Circumference. Figure 8.57 shows a circle created with the **CIRCLE** command and three digitizes (**D1**, **D2**, and **D3**), which define three points on the circle's circumference. The following command was used:

> `Command:` CIRCLE
> `3P/2P/TTR/<Center point>: 3P` (pick the three-point option)
> `First point on circumference:` D1
> `Second point on circumference:` D2
> `Third point on circumference:` D3

Drawing Circles by Digitizing the Diameter. CAD systems can create a circle by calculating the circle's diameter from the distance between two digitized locations. Figure 8.58 illustrates a circle created with the **CIRCLE** command and two digitized points (**D1** and **D2**), which established opposite points (the diameter) on the circle's circumference.

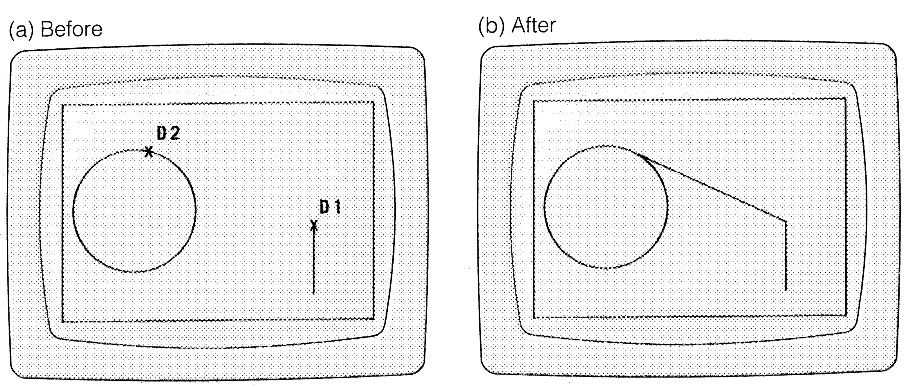


FIGURE 8.56 Drawing a Line Tangent to a Circle and at the End of an Existing Line

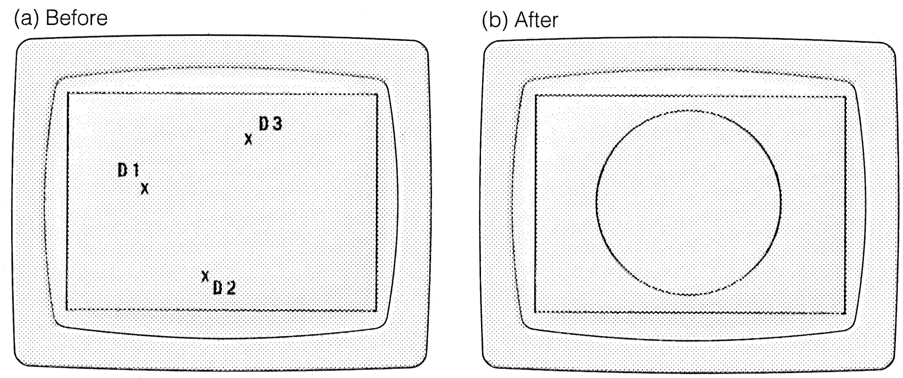


FIGURE 8.57 Drawing a Three-Point Circle

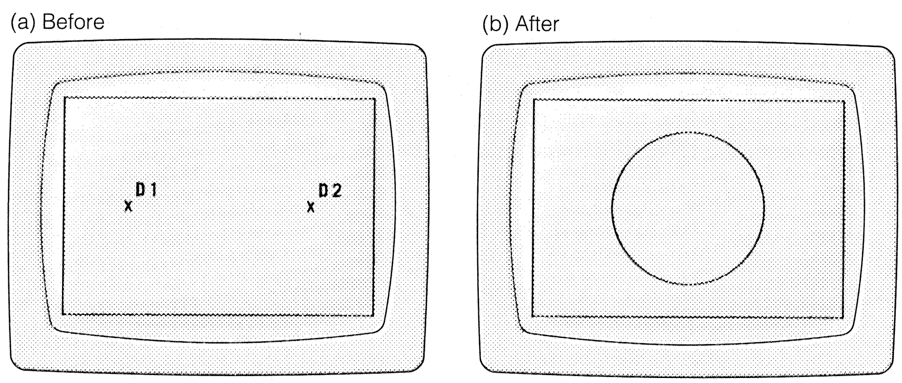


FIGURE 8.58 Drawing a Circle by Digitizing a Diameter

The following command was used:

```
Command: CIRCLE
3P/2P/TTR/<Center point>: 2P (pick the two-point option)
First point on diameter: D1
Second point on diameter: D2
```

Drawing Circles by Digitizing the Radius. CAD systems can create a circle by calculating the circle's radius from two digitized locations. In Figure 8.59, the **CIRCLE** command was selected along with two digitized locations. Point **D1** established the circle's center, and **D2** identified a point on the circumference. The following command was used:

```
Command: CIRCLE
3P/2P/TTR/<Center point>: D1 (pick the center of the circle)
Diameter/<Radius>: R (pick radius option)
Radius: D2 (digitize location)
```

Specifying a Diameter or a Radius. You can also specify a circle's diameter or radius by entering an explicit value. Only one digitized point is required for creating a circle when the diameter or the radius value is known (Fig. 8.60).

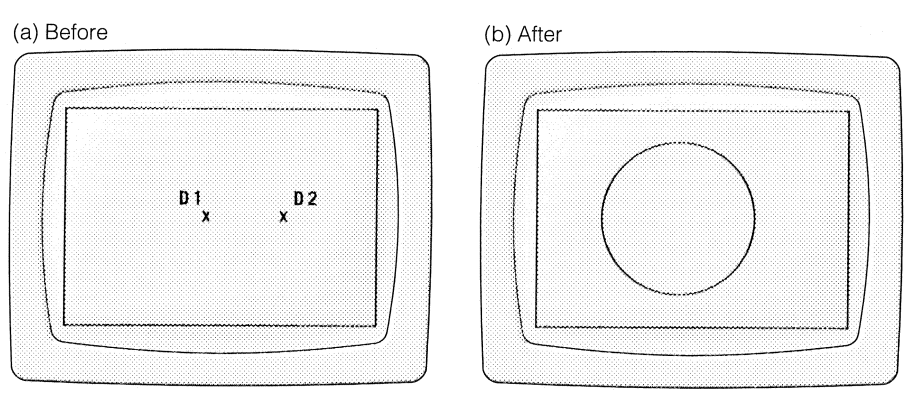


FIGURE 8.59 Drawing a Circle by Digitizing a Radius

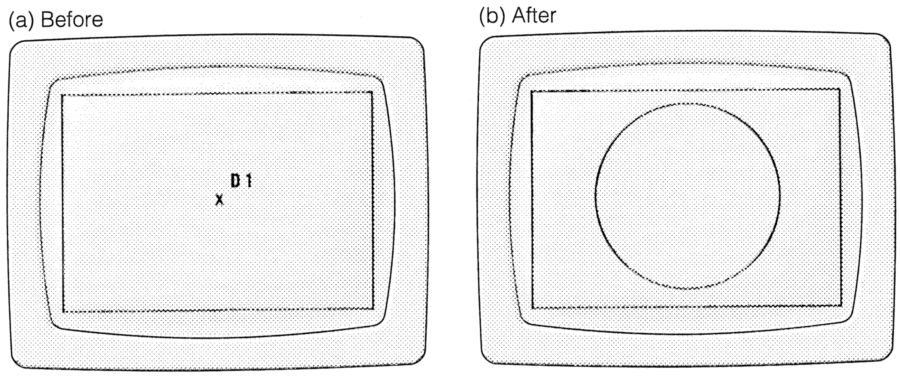


FIGURE 8.60 Drawing a Circle by Specifying the Radius Value

The following command was used:

Command: CIRCLE
3P/2P/TTR/<Center of circle>: D1 (center of circle)
Diameter/<Radius>: 3.00 (radius value)

8.9.4 Arcs

There are many ways to create an arc, including specifying the start point, specifying a point on the arc's path, and specifying the endpoint. The system creates the arc in the direction in which the points were digitized. AutoCAD lets you create an arc many different ways, only two of which we will present here.

Drawing Arcs by Digitizing Three Locations. Figure 8.61 shows an arc created using the **ARC** command and three digitizes. Point **D1** established the arc's starting point, **D2** is a point on the arc, and **D3** is the endpoint. The following command was used:

Command: ARC
Center/::Start point>: D1
Center/End/Second point>: D2
Endpoint: D3

Drawing Arcs by Specifying the Center, the Starting Point, and the Endpoint. Figure 8.62 illustrates an arc that was created using the **ARC** command along with specifying the center point, the starting point, and the endpoint. The following command was used:

Command: ARC
Center/<Start point>: C (pick center option)
of D1 (digitize center of arc)
Start point: D2 (digitize start point)
Angle/Length/<Endpoint>: D3 (digitize the endpoint of the arc)

Drawing Tangent Arcs. A tangent arc is also called a *fillet*. A fillet is an arc created tangent to existing geometry. The **FILLET** command is used to construct tangent arcs. A fillet of a specified size is created by entering the diameter or the radius value. Fillets may be created with respect to points, lines, circles, and arcs. In Figure 8.63, the **FILLET** command was specified and the two lines were picked. In Figure 8.64, the following command was used to create the fillet:

Command: FILLET
Polyline/Radius/<Select first object>: R (select R for radius)
Enter fillet radius: 1.50 (set the radius default)
Command: <RETURN> (reenter the command)
FILLET Polyline/Radius/<Select first object>: D1 (pick first entity)
Select second object: D2 (pick second entity)

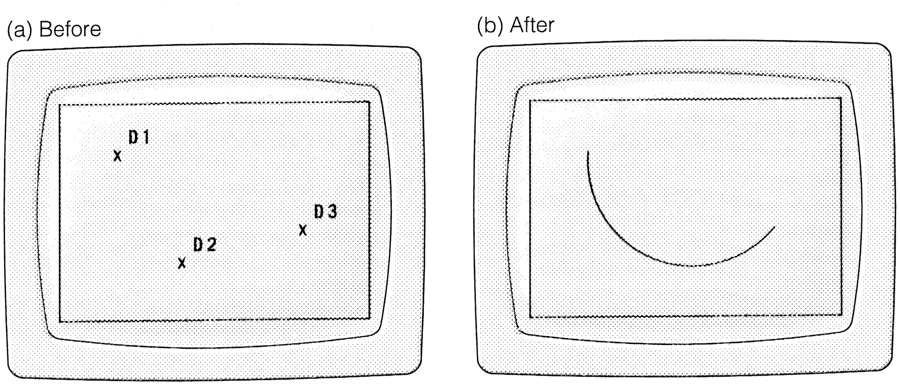


FIGURE 8.61 Drawing an Arc Through Three Digitized Points

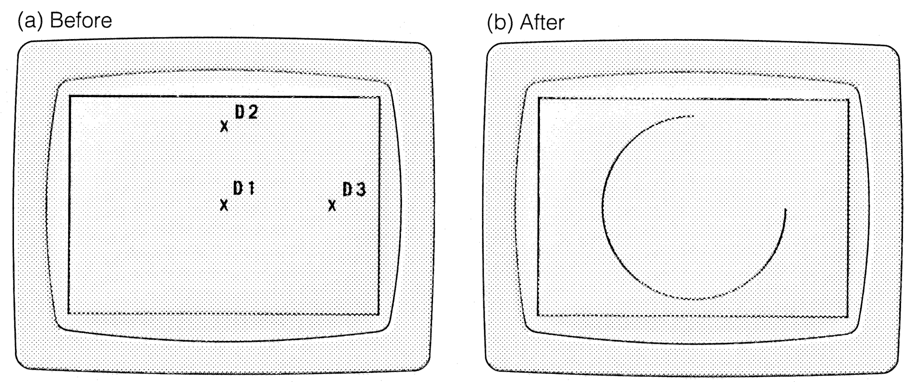


FIGURE 8.62 Drawing an Arc

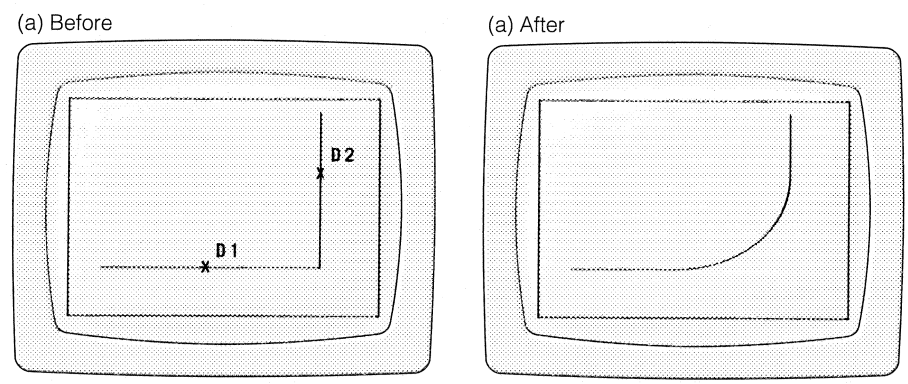


FIGURE 8.63 Drawing a Fillet Between Two Lines

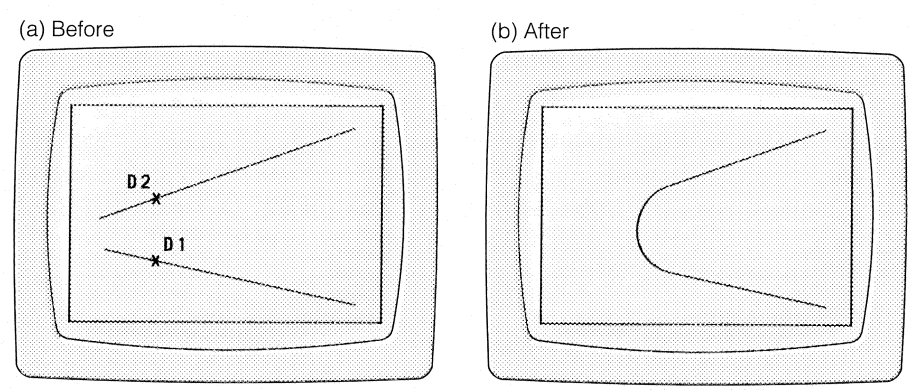


FIGURE 8.64 Drawing a Fillet

8.9.5 Drawing Ellipses

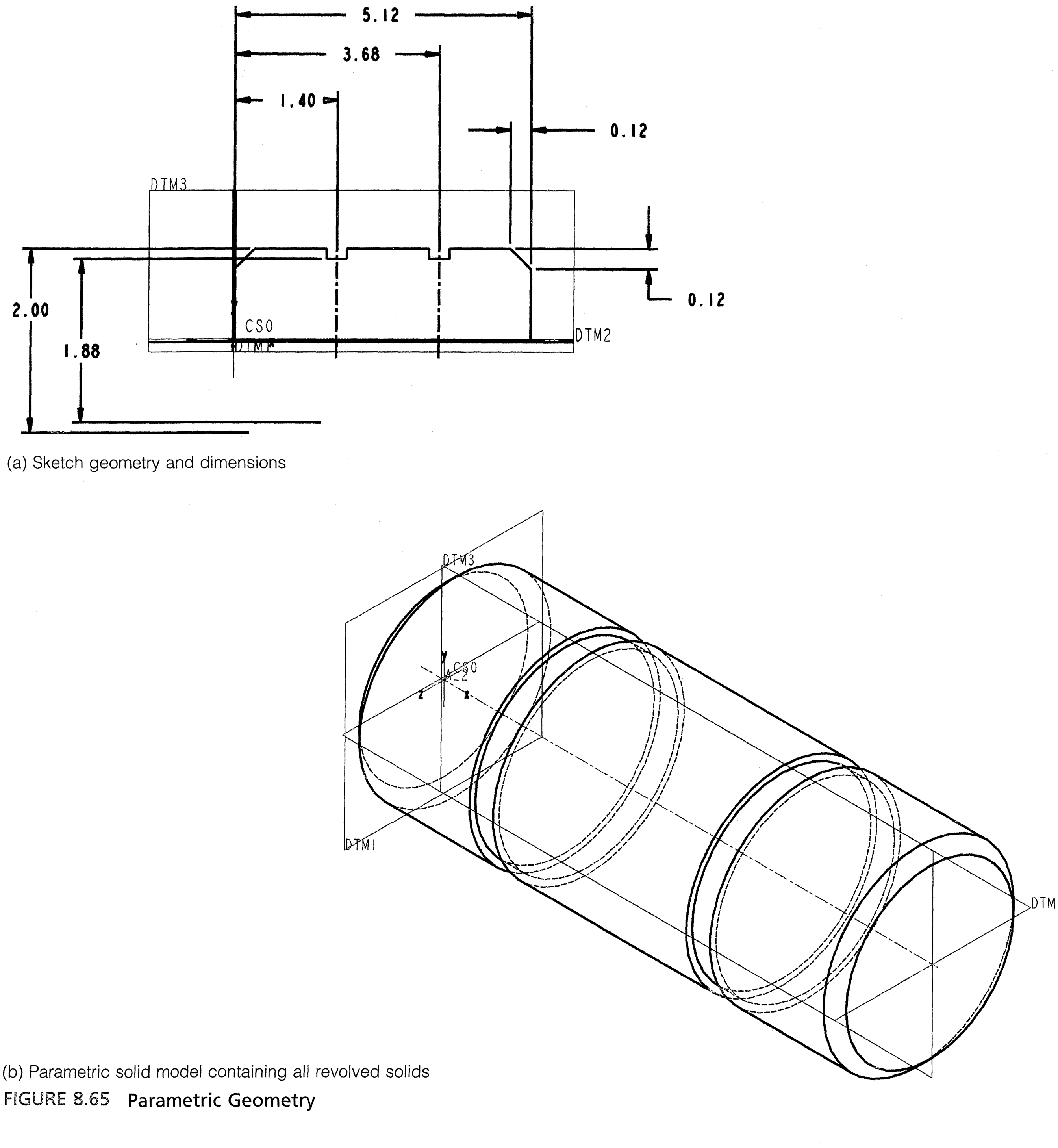
An ellipse is an elongated circle. Mathematically, it is defined as a cone intersected by a plane and therefore can be referred to as a conic. CAD systems draw perfect ellipses. In the following example, the **ELLIPSE** command is used to create an ellipse with a major axis of 50 mm and a minor axis of 40 mm.

`Command:` **ELLIPSE**
`<Axis endpoint 1>/Center: C` (pick C for center)
`Center of ellipse:` (digitize location for center of ellipse)
`Axis endpoint: @25>0.00` (pick polar point for half the major axis at 0°)
`<Other axis endpoint>/Rotation:`
`@ 20< 270` (pick polar point for half the minor axis at 270°)

8.9.6 Drawing Polygons

Many CAD systems have a command that allows the automatic creation of a polygon. With AutoCAD the **POLYGON** command can be used to draw a polygon of any number of sides and of any size. The following command creates a hexagon with distance across the flats of 1.00 in.:

`Command:` **POLYGON**
`Number of sides: 6` (enter number of sides)
`Edge of/<Center of Polygon>:` (digitize center of polygon)
`Inscribed in circle/Circumscribed about circle: C` (pick circumscribed)
`Radius of circle:` **1.00** (distance across flats)



(a) Sketch geometry and dimensions

(b) Parametric solid model containing all revolved solids

FIGURE 8.65 Parametric Geometry

It is important for an engineer to master the manual procedures and techniques introduced in this chapter. Many of the manual construction techniques are needed when drafting and designing projects in industry, even when a CAD system is available. In Figure 8.65(a) and (b) we can see how the part's design was created by sketching a simple geometric shape (parametric modeling) of the outline (section) and adding dimensions. The pin was created by revolving the section 360° about a sketched centerline. Because of the simplicity of parametric modeling, the pin was modeled in less than 3 minutes!

QUIZ

True or False

1. Both hyperbolas and cones are generated from conic sections
2. In helix construction, the distance traveled by one point for one revolution measured parallel to the axis is called the lead.
3. The concentric method of ellipse construction is more accurate than using a template.
4. Tangent arcs are basically the same thing as fillets.
5. Squares, hexagons, pentagons, and ellipses are regular polygons.
6. A circle can be used to construct all forms of regular polygons.
7. A bisector of a line or an angle divides the line or angle into an equal number of parts.
8. A regular polygon has equal angles.

Fill in the Blanks

9. All geometric forms are composed of __________ and their __________.
10. __________ are used to divide lines into __________ __________ parts.
11. An __________ circle of a triangle will touch all __________ sides.
12. The distance across the __________ of an octagon will be __________ to the diameter of the __________ circle.
13. To __________ a circle means to __________ out its circumference along a straight line.
14. A __________ arc is a curve connecting two entities and is also known as a __________.
15. __________ __________ are equally spaced along their entire length.
16. Geometric forms include: __________, __________, __________, and __________.

Answer the Following

17. When is it appropriate to use the four-center method of ellipse construction?
18. Give a simple definition of a line and a curve.
19. What are the five types of figures that result from the intersection of a cone and a plane?
20. Define a cylindrical helix and a conical helix.
21. Why would the graphical method of dividing a line be more accurate than the mathematical method?
22. Name four solid shapes commonly used in industry.
23. Why is it important to learn the manual method of constructing geometric forms such as ellipses instead of just using templates?
24. Describe an industrial application of geometric construction.

EXERCISES

Exercises may be assigned as sketching, instrument, or CAD projects. Transfer the given information to an "A"-size sheet of .25 in. grid paper. Complete all views, and solve for proper visibility, including centerlines, object lines, and hidden lines. Exercises that are not assigned by the instructor can be sketched in the text to provide practice and to enhance understanding of the preceding instructional material.

After Reading the Chapter Through Section 8.5.7, You May Complete Exercises 8.1 Through 8.4

Exercise 8.1(A) Bisect the line, the angle, and the arc.

Exercise 8.1(B) Divide line 1 into eleven equal parts using the graphical method. Divide line 2 into seven equal parts and line 3 into proportional parts having ratios of 3:2:5.

Exercise 8.1(C) Construct a hexagon inside and an octagon around the outside of the given circle.

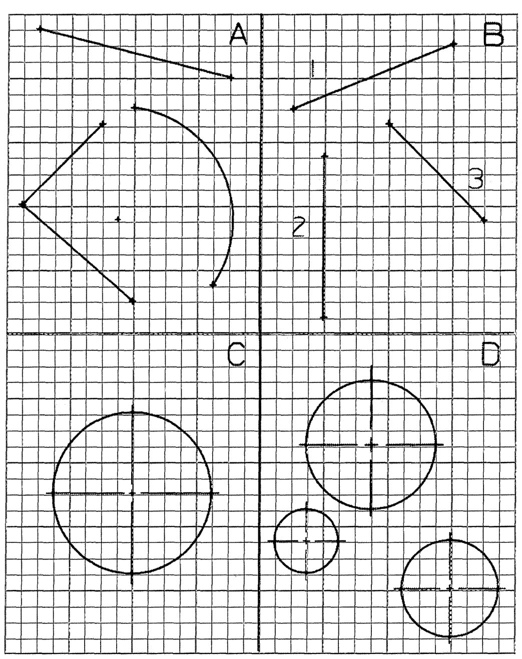
Exercise 8.1(D) Draw every possible tangency for the three circles.

Exercise 8.2(A) Draw a 40 mm radius arc (fillet) between the connected lines. Connect the two lines with a tangent arc (fillet) using a 25 mm radius.

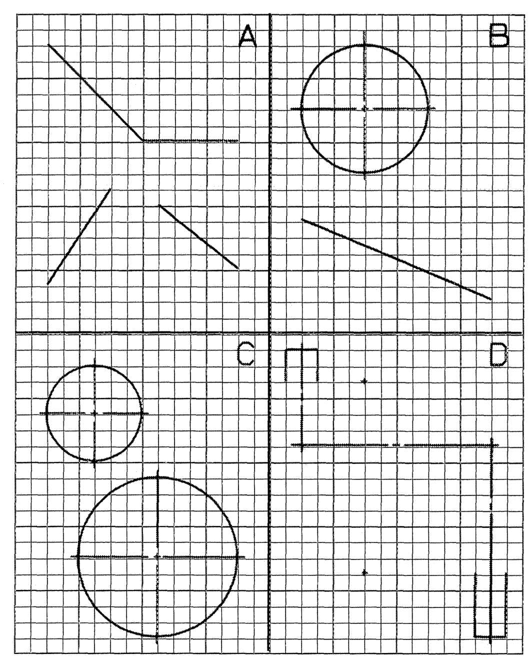
Exercise 8.2(B) Draw a 2 in. radius or a 50 mm radius arc (fillet) between the circle and the line.

Exercise 8.2(C) Construct a 3 in. or a 70 mm inside arc (fillet) on the top right side of the two circles. Draw a 5 in. or a 120 mm outside (enclosing) arc connecting the two circles on the bottom left.

Exercise 8.2(D) Draw an S-shaped curve using the given lines and points.



EXERCISE 8.1



EXERCISE 8.2

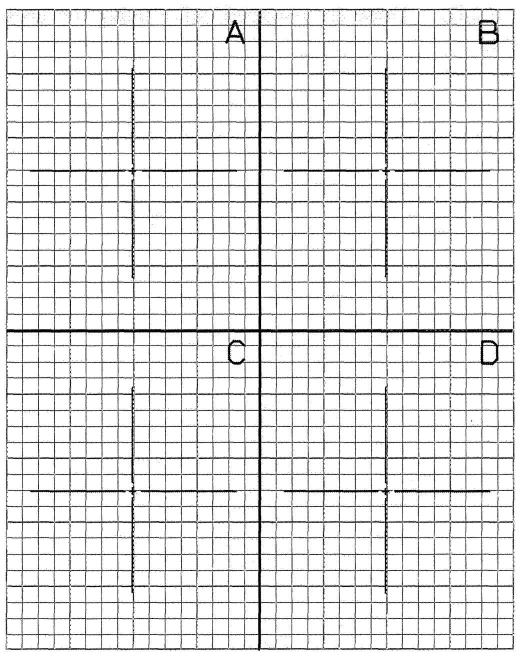
Exercise 8.3(A) Given two circles of 2.25 and 3.75 in., draw an ellipse via the concentric-circle method. For a metric problem, use diameters of 70 and 90 mm.

Exercise 8.3(B) Given a major diameter of 4.75 in. and a minor diameter of 3.50 in., draw an ellipse using the four-center method. Draw the ellipse so that the major diameter is vertical. Use 120 and 80 mm for a metric problem.

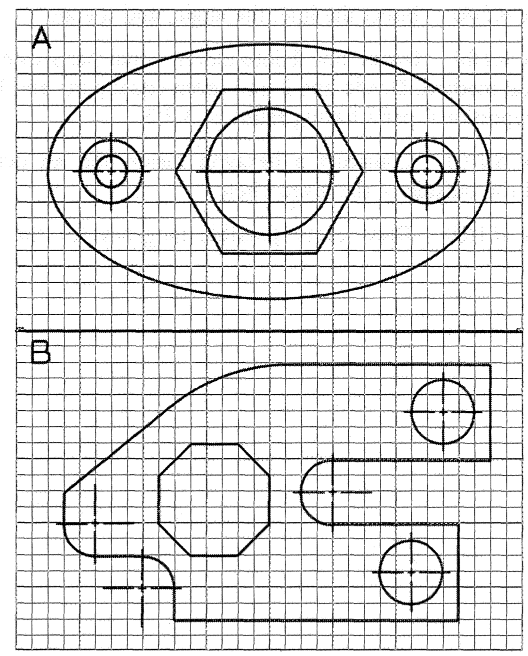
Exercise 8.3(C) Given circles 2.50 and 3.5 in. in diameter, draw an ellipse via the approximate method. Use 60 and 80 mm for a metric problem.

Exercise 8.3(D) Draw two identical ellipses. Use the concentric-circle method for one and the four-center method for the other. Make 4.5 in. the vertical diameter and 2.75 in. the minor (horizontal in this case) diameter. If metrics are selected as the unit of measurement, use 110 and 70 mm. Compare the two methods for quality and accuracy.

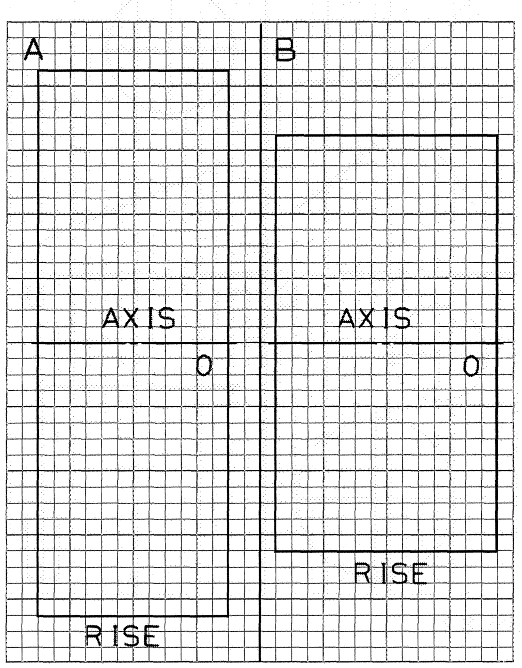
Exercise 8.4(A) and (B) Draw the two figures using geometric construction techniques covered in the chapter.



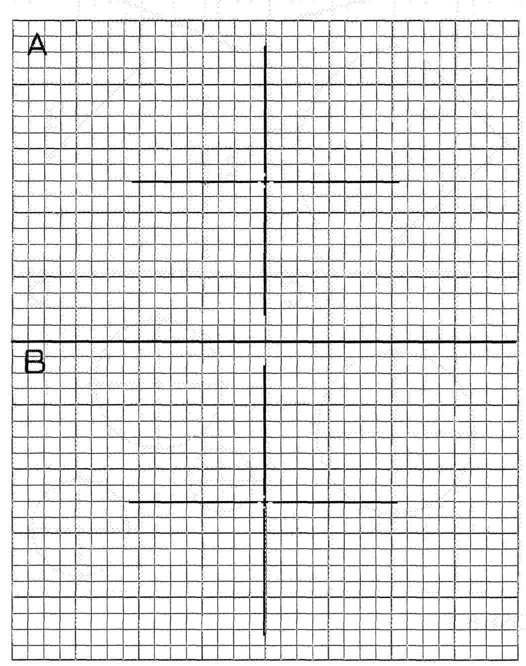
EXERCISE 8.3



EXERCISE 8.4



EXERCISE 8.5



EXERCISE 8.6

After Reading the Chapter Through Section 8.7.1, You May Complete the Following Exercises

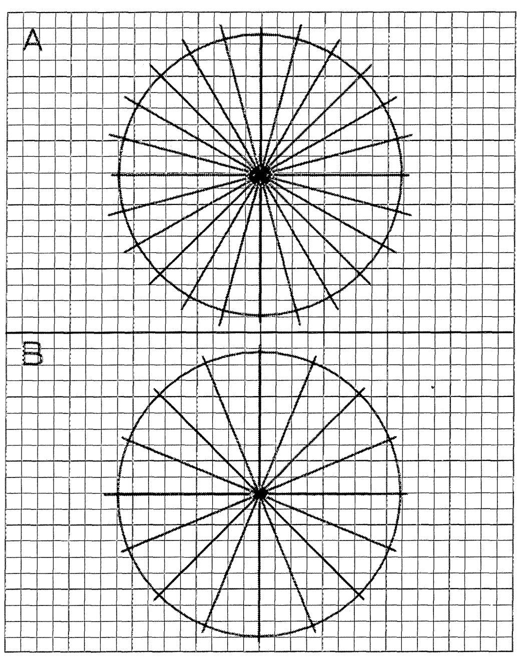
Exercise 8.5(A) and (B) Given the rectangle, the rise, and the axis, draw a parabola for each of the problems.

Exercise 8.6(A) or (B) Draw a hyperbola using a 4 in. (or 100 mm) diameter for the cone base and a height of 4 in. (or 100 mm). Pass a cutting plane vertically through the cone 1 in. (or 25 mm) to the right of the cone's vertical axis. For (B) use 5 in. (or 120 mm) as the base diameter and 4.5 in. (or 110 mm) as the cone's height. Draw the cutting plane vertically through the cone at 1.35 in. (or 35 mm) to the left of the cone's axis. Only one of these two problems can be done on the exercise page since the opposite space will be needed for construction.

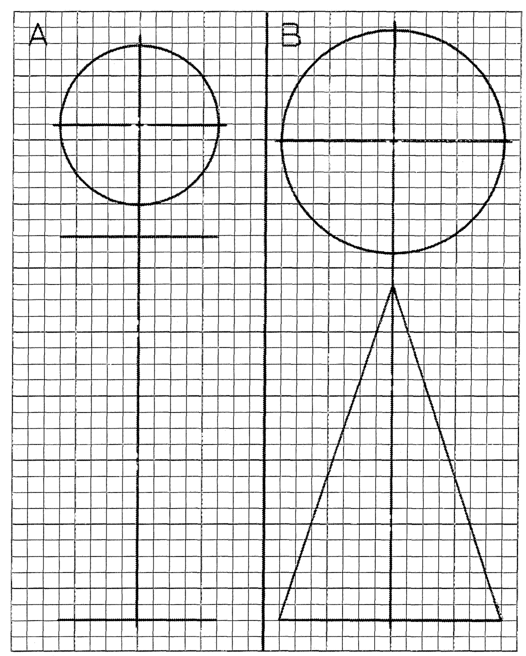
Exercise 8.7(A) and (B) Construct a spiral of Archimedes. Start at the center and use a line [vertical in (A) and horizontal in (B)] as the beginning line for marking off the divisions. Draw the spiral clockwise in A and counterclockwise in B.

Exercise 8.8(A) Using the given cylinder for the diameter and the height, draw a right-handed helix with a lead of 3 in. Start the helix at the middle of the cylinder at the base where the axis line crosses the baseline

Exercise 8.8(B) Use the given cone diameter and height to construct a left-handed conical helix. Start the helix on the lower left of the base line. Use a lead of 2.5 in.



EXERCISE 8.7

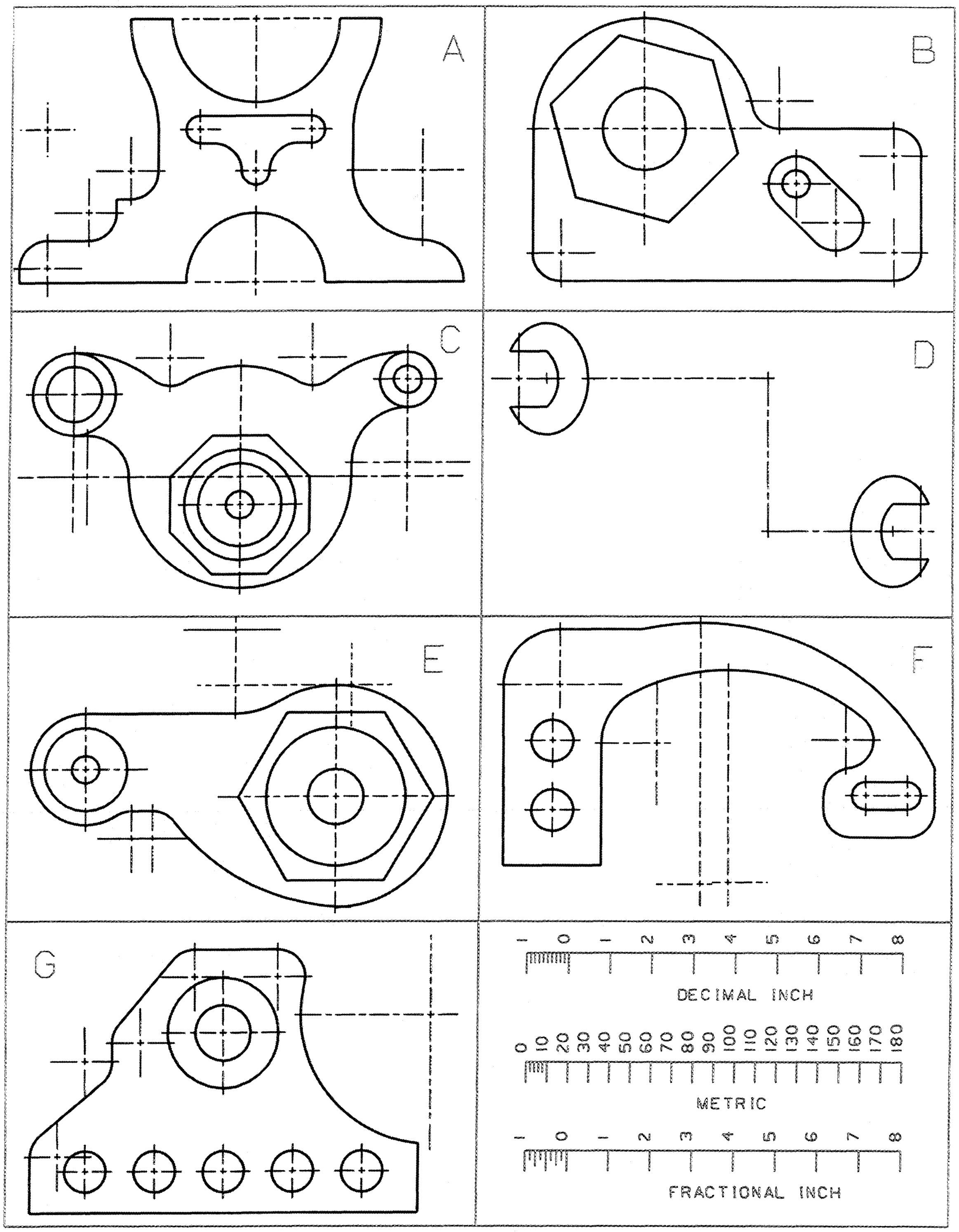


EXERCISE 8.8

PROBLEMS

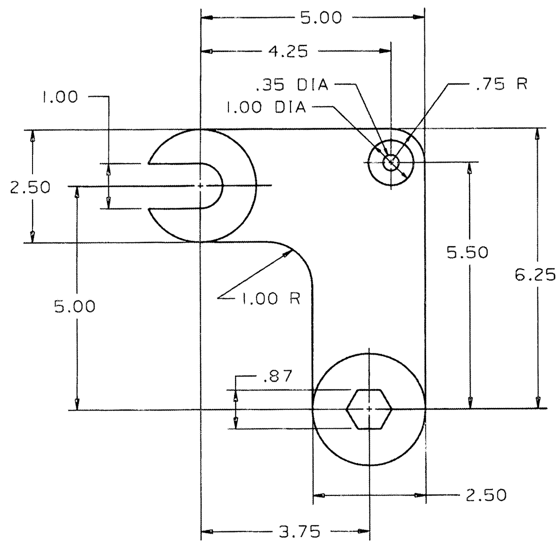
Dimensions are provided for construction of each figure. They are not to be considered correct per ANSI standards. Do not dimension without the instructor's approval.

Problems 8.1 (A) through (G) Transfer the problems to an appropriate-size drawing format (one per drawing). Use one of the three scales provided. For Problem 8.1(D) the handle of the wrench is to be 1.50 in. (or 40 mm) in width throughout the ogee curve.

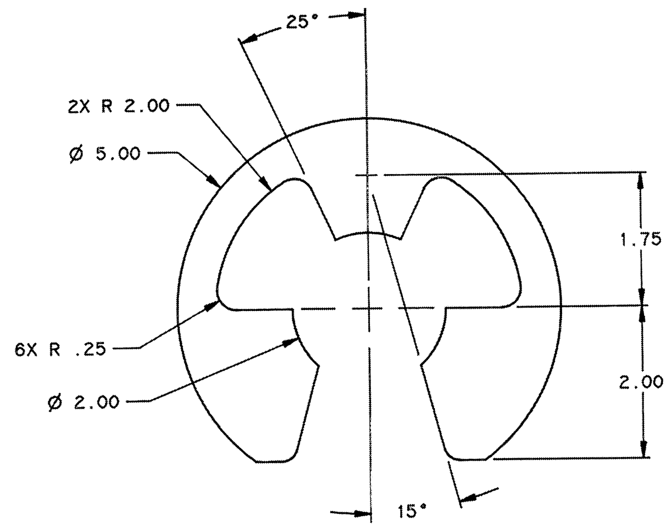


PROBLEM 8.1

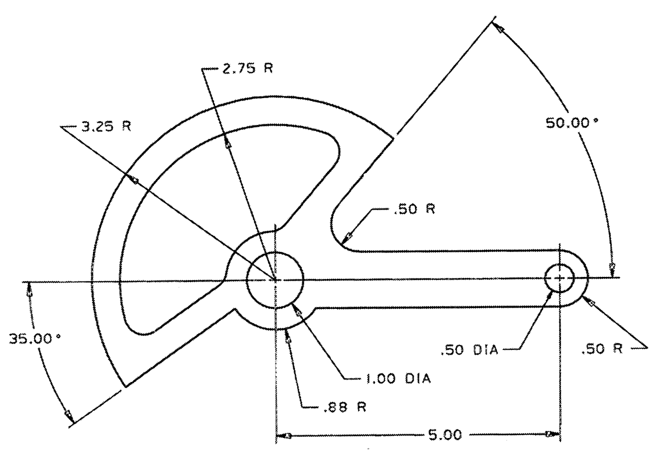
Problems 8.2 through 8.13 Draw each of the assigned problems on separate sheets. For Problem 8.6 the 7.500 and the 8.00 radius arcs can be located by extending the centerline of the slot vertically.



PROBLEM 8.2

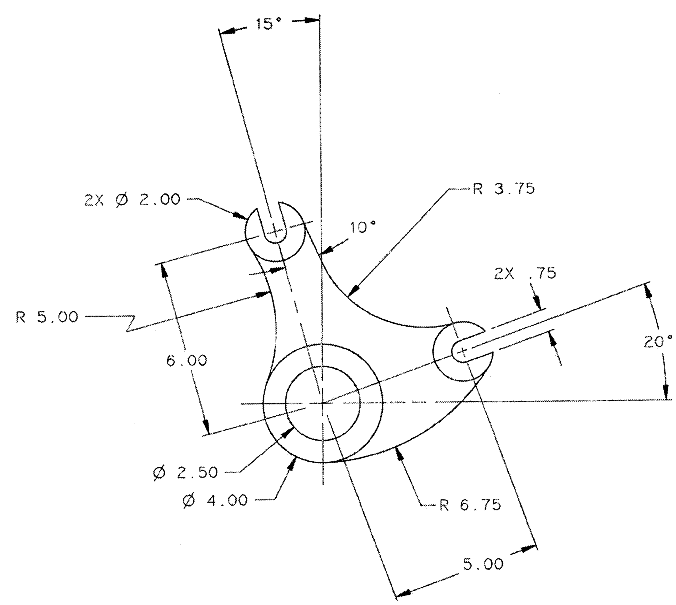


PROBLEM 8.4

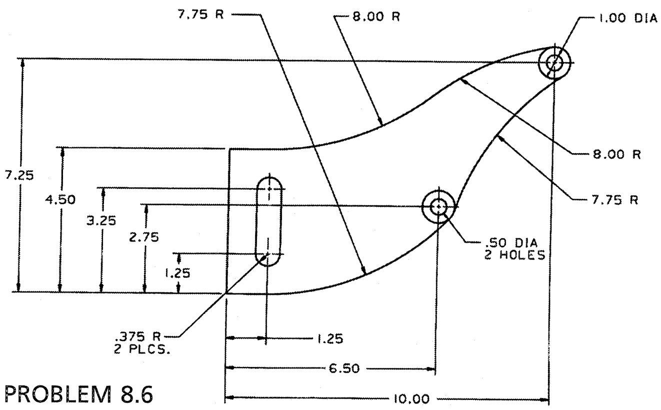


ALL FILLETS .35 R UNLESS OTHERWISE SPECIFIED

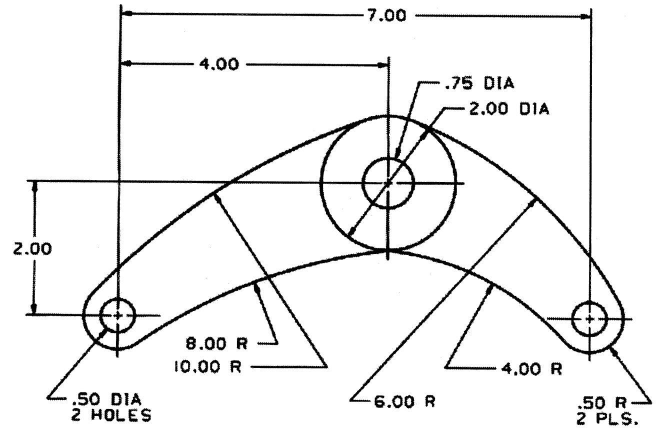
PROBLEM 8.3



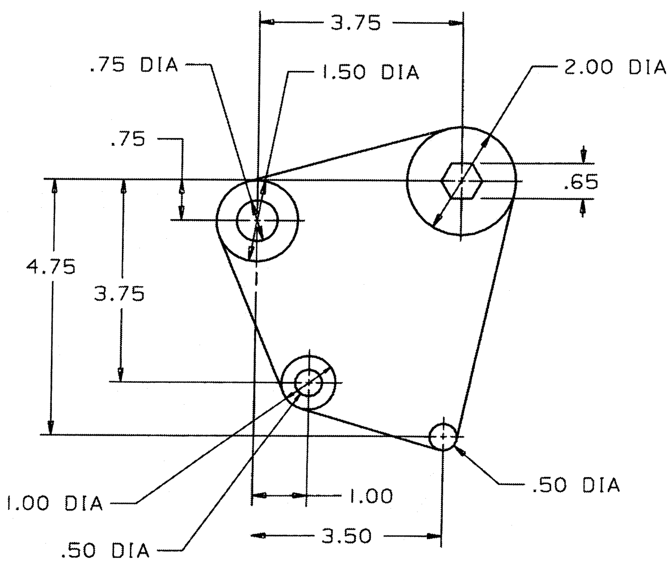
PROBLEM 8.5



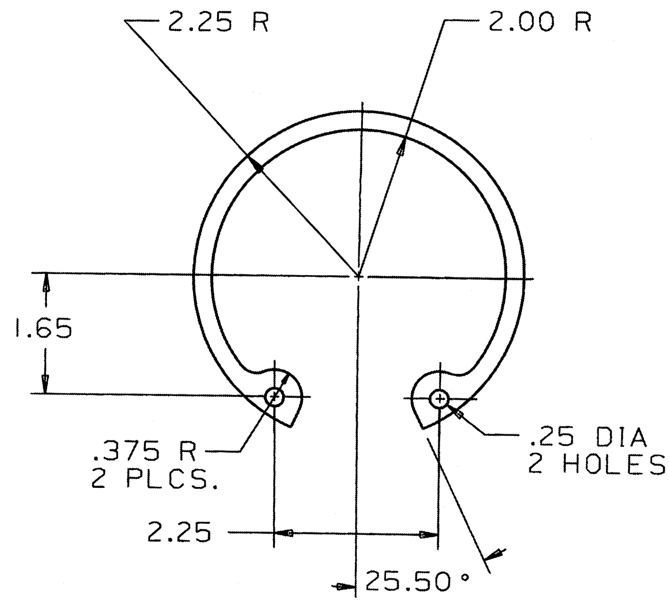
PROBLEM 8.6



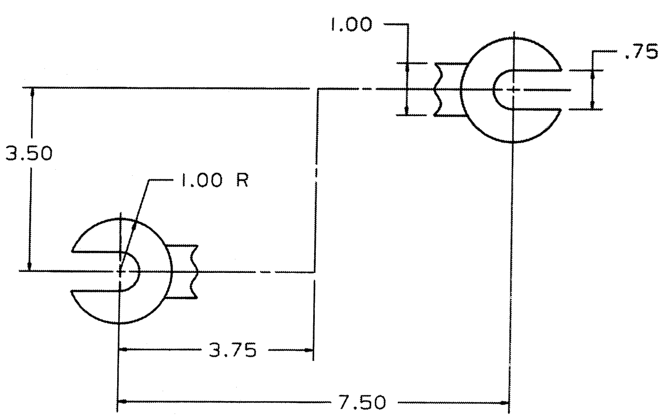
PROBLEM 8.9



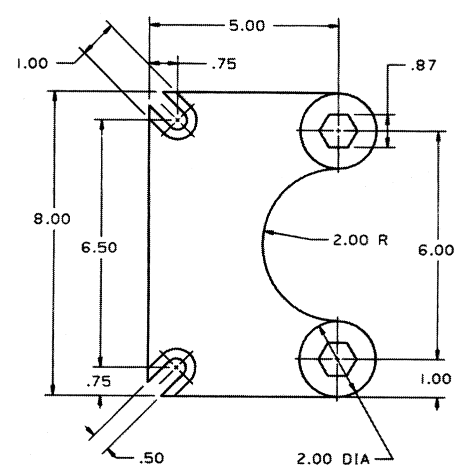
PROBLEM 8.7



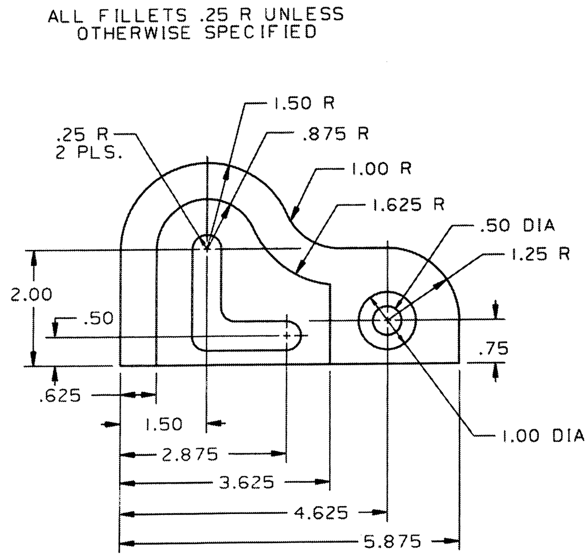
PROBLEM 8.10



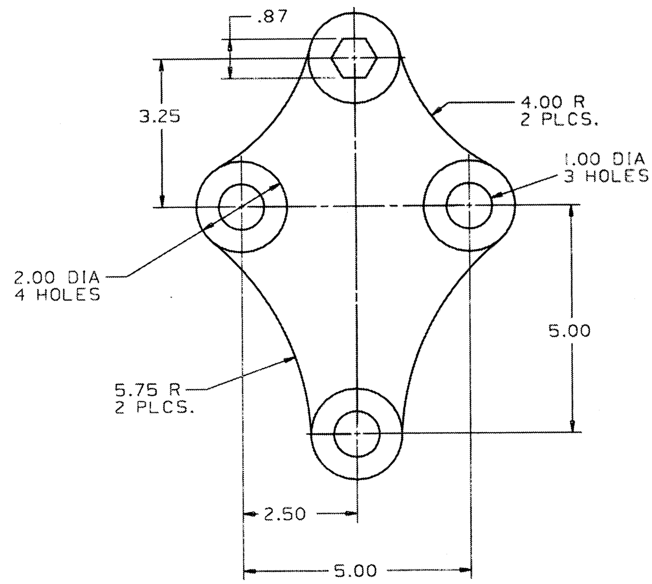
PROBLEM 8.8



PROBLEM 8.11

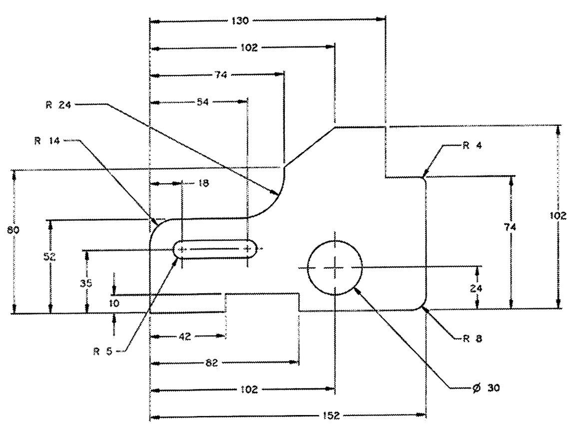


PROBLEM 8.12

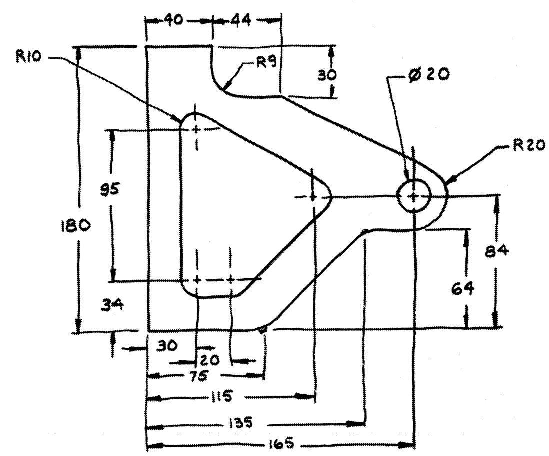


PROBLEM 8.13

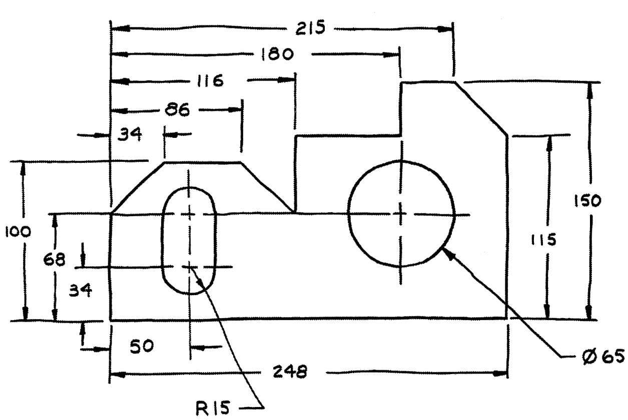
Problems 8.14 through 8.24 Draw each of the projects on appropriate-size drawing format. These problems use metric measurements. Draw only the front view for Problems 8.23 and 8.24.



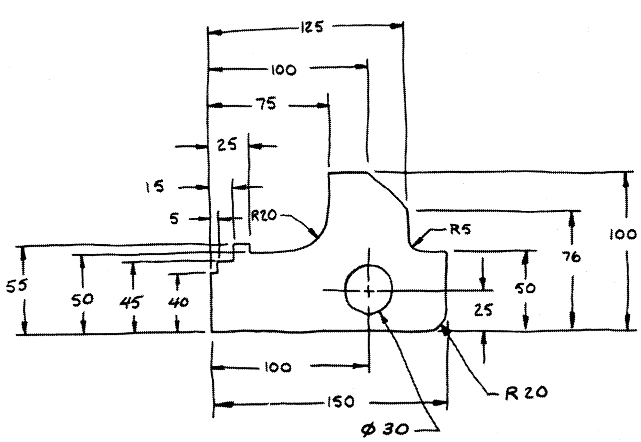
PROBLEM 8.14



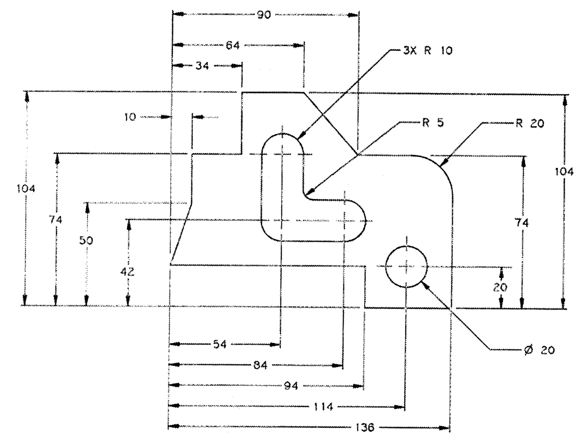
PROBLEM 8.16



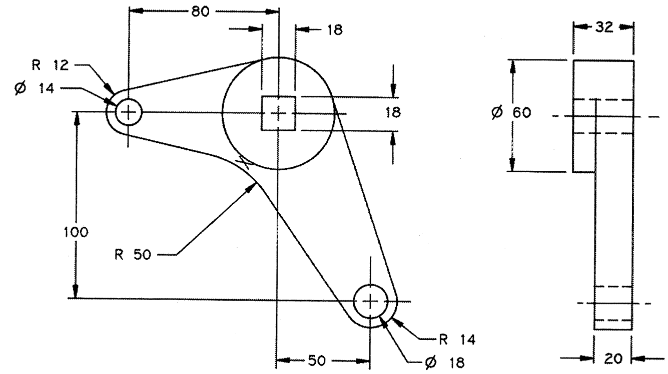
PROBLEM 8.15



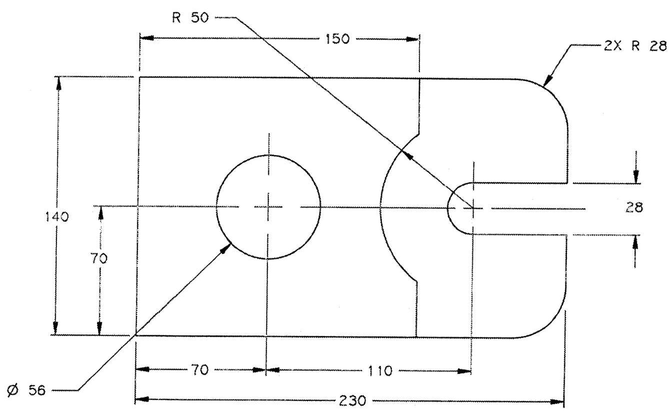
PROBLEM 8.17



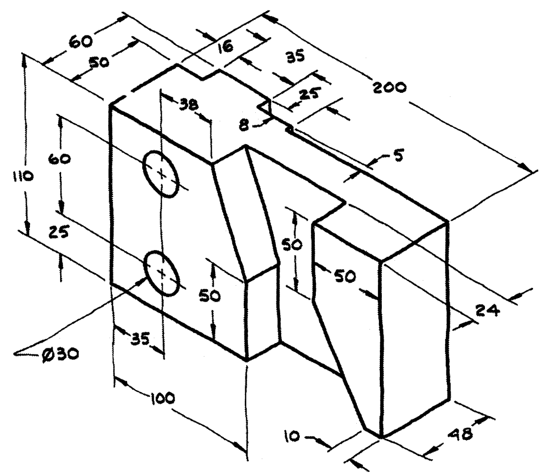
PROBLEM 8.18



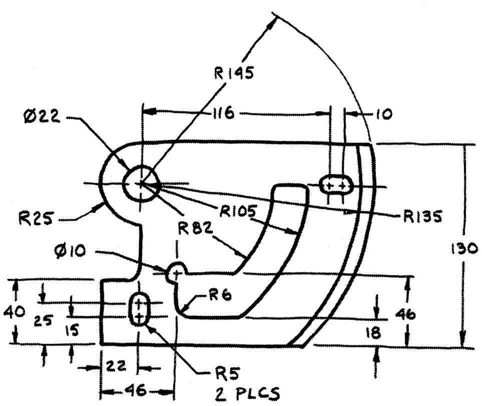
PROBLEM 8.22



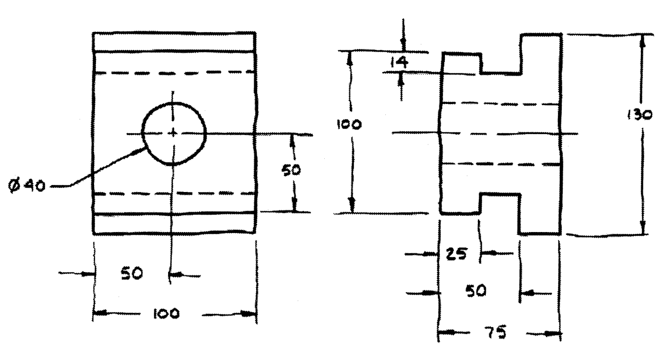
PROBLEM 8.19



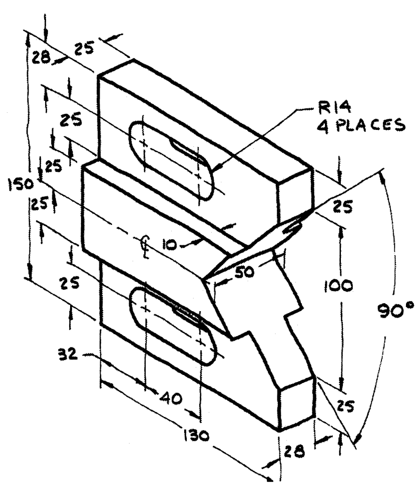
PROBLEM 8.23



PROBLEM 8.20



PROBLEM 8.21



PROBLEM 8.24

Problem 8.25 Divide a 5 in. (120 mm) line into five equal parts.

Problem 8.26 Construct bisectors of the angles of a triangle having a ratio of 3:5:6 units for the sides. Use centimeters or inches as the units.

Problem 8.27 Bisect a 55° angle.

Problem 8.28 Draw a triangle having sides with the proportions 3:4:5.

Problem 8.29 Draw a hexagon that is 3 in. (or 70 mm) across the flats.

Problem 8.30 Draw a hexagon that is 75 mm across the corners.

Problem 8.31 Construct a seven-sided regular polygon in a 5 in. (120 mm) diameter circle.

Problem 8.32 Find the center of a 4 in. (or 100 mm) circle.

Problem 8.33 Connect two lines forming a 35° angle with a radius arc of 1.00 in. (or 25 mm).

Problem 8.34 Connect two perpendicular lines with a radius arc of l.00 in. (or 30 mm).

Problem 8.35 Draw an ellipse having a major axis of 70 mm and a minor axis of 50 mm.

Problem 8.36 Construct the inscribed circle of a 2 × 3.5 × 4 unit triangle.

Problem 8.37 Draw a circumscribed-circle triangle. Use centimeters as units.

Problem 8.38 Find the center of a 4 in. (or 100 mm) circle by perpendicular bisectors. Rectify the circle.

Problem 8.39 Draw a cylindrical helix having an 80 mm diameter base, a height of 140 mm, and a lead of 50 mm. Draw it as a right-handed helix.

Problem 8.40 Draw a left-handed conical helix with a base diameter of 3 in. (or 70 mm), a height of 4.5 in. (or 110 mm), and a lead of 2.5 in. (or 60 mm).

Problem 8.41 Draw a spiral of Archimedes using a 5 in. (or 120 mm) diameter with angles of 30° and .125 in. (or 10 mm) divisions.